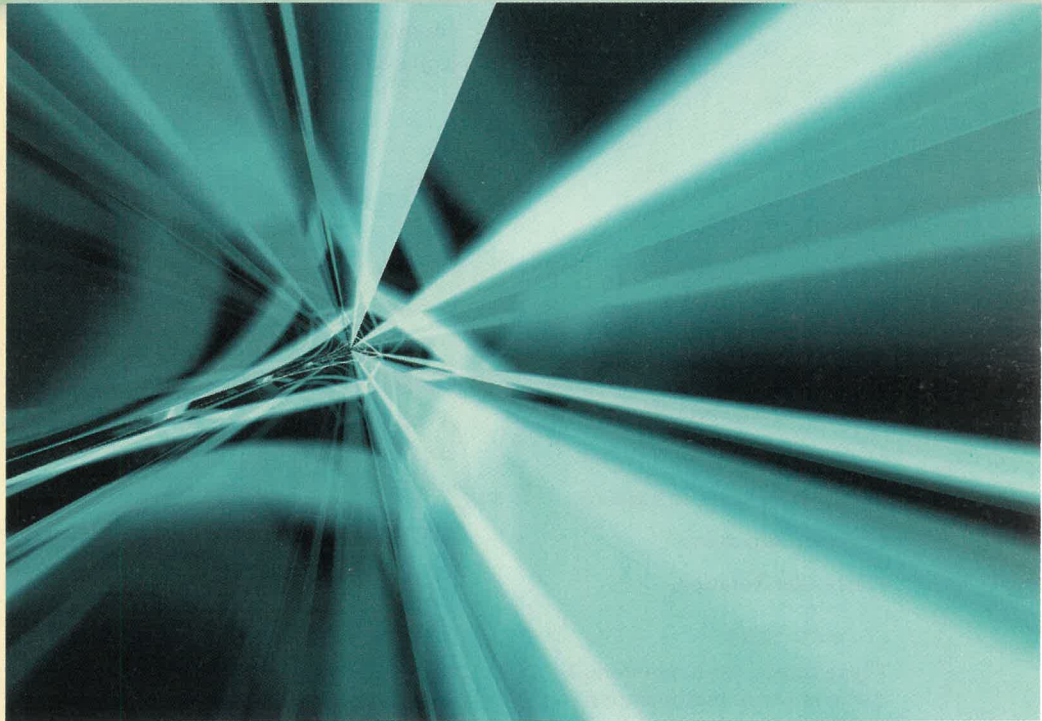


Prerequisites

- P.1** Real Numbers
- P.2** Cartesian Coordinate System
- P.3** Linear Equations and Inequalities
- P.4** Lines in the Plane
- P.5** Solving Equations Graphically, Numerically, and Algebraically
- P.6** Complex Numbers
- P.7** Solving Inequalities Algebraically and Graphically



Large distances are measured in *light years*, the distance light travels in one year. Astronomers use the speed of light, approximately 186,000 miles per second, to approximate distances between planets. See page 39 for examples.

Chapter P Overview

Historically, algebra was used to represent problems with symbols (algebraic models) and solve them by reducing the solution to algebraic manipulation of symbols. This technique is still important today. Graphing calculators are used today to approach problems by representing them with graphs (graphical models) and solve them with numerical and graphical techniques of the technology.

We begin with basic properties of real numbers and introduce absolute value, distance formulas, midpoint formulas, and write equations of circles. Slope of a line is used to write standard equations for lines and applications involving linear equations are discussed. Equations and inequalities are solved using both algebraic and graphical techniques.

P.1 Real Numbers

What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

... and why

These topics are fundamental in the study of mathematics and science.

Representing Real Numbers

A **real number** is any number that can be written as a decimal. Real numbers are represented by symbols such as -8 , 0 , 1.75 , $2.333\dots$, $0.\overline{36}$, $8/5$, $\sqrt{3}$, $\sqrt[3]{16}$, e , and π .

The set of real numbers contains several important subsets:

The **natural (or counting) numbers**: $\{1, 2, 3, \dots\}$

The **whole numbers**: $\{0, 1, 2, 3, \dots\}$

The **integers**: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

The braces $\{ \}$ are used to enclose the **elements**, or **objects**, of the set. The rational numbers are another important subset of the real numbers. A **rational number** is any number that can be written as a ratio a/b of two integers, where $b \neq 0$. We can use **set-builder notation** to describe the rational numbers:

$$\left\{ \frac{a}{b} \mid a, b \text{ are integers, and } b \neq 0 \right\}$$

The vertical bar that follows a/b is read “such that.”

The decimal form of a rational number either **terminates** like $7/4 = 1.75$, or is **infinitely repeating** like $4/11 = 0.363636\dots = 0.\overline{36}$. The bar over the 36 indicates the block of digits that repeats. A real number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating. For example, $\sqrt{3} = 1.7320508\dots$ and $\pi = 3.14159265\dots$

Real numbers are approximated with calculators by giving a few of its digits. Sometimes we can find the decimal form of rational numbers with calculators, but not very often.

1/16	.0625
55/27	2.037037037
1/17	.0588235294

FIGURE P.1 Calculator decimal representations of $1/16$, $55/27$, and $1/17$ with the calculator set in floating decimal mode. (Example 1)

EXAMPLE 1 Examining Decimal Forms of Rational Numbers

Determine the decimal form of $1/16$, $55/27$, and $1/17$.

SOLUTION Figure P.1 suggests that the decimal form of $1/16$ terminates and that of $55/27$ repeats in blocks of 037.

$$\frac{1}{16} = 0.0625 \quad \text{and} \quad \frac{55}{27} = 2.\overline{037}$$

We cannot predict the *exact* decimal form of $1/17$ from Figure P.1, however we can say that $1/17 \approx 0.0588235294$. The symbol \approx is read “*is approximately equal to*.” We can use long division (see Exercise 66) to show that

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

Now try Exercise 3.

The real numbers and the points of a line can be matched *one-to-one* to form a **real number line**. We start with a horizontal line and match the real number zero with a point O , the **origin**. **Positive numbers** are assigned to the right of the origin, and **negative numbers** to the left, as shown in Figure P.2.

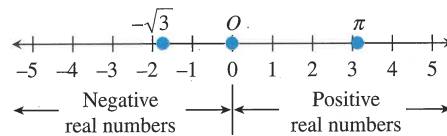


FIGURE P.2 The real number line.

Every real number corresponds to one and only one point on the real number line, and every point on the real number line corresponds to one and only one real number. Between every pair of real numbers on the number line there are infinitely many more real numbers.

The number associated with a point is **the coordinate of the point**. As long as the context is clear, we will follow the standard convention of using the real number for both the name of the point and its coordinate.

Order and Interval Notation

The set of real numbers is **ordered**. This means that we can compare any two real numbers that are not equal using inequalities and say that one is “less than” or “greater than” the other.

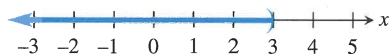
UNORDERED SYSTEMS

Not all number systems are ordered. For example, the complex number system, to be introduced in Section P.6, has no natural ordering.

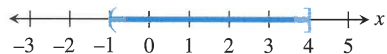
OPPOSITES AND NUMBER LINE

$$a < 0 \Rightarrow -a > 0$$

If $a < 0$, then a is to the left of 0 on the real number line, and its opposite, $-a$, is to the right of 0. Thus, $-a > 0$.

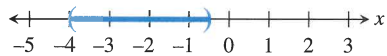


(a)



(b)

-0.5



(c)



(d)

FIGURE P.3 In graphs of inequalities, parentheses correspond to $<$ and $>$ and brackets to \leq and \geq . (Examples 2 and 3)

Order of Real Numbers

Let a and b be any two real numbers.

Symbol	Definition	Read
$a > b$	$a - b$ is positive	a is greater than b
$a < b$	$a - b$ is negative	a is less than b
$a \geq b$	$a - b$ is positive or zero	a is greater than or equal to b
$a \leq b$	$a - b$ is negative or zero	a is less than or equal to b

The symbols $>$, $<$, \geq , and \leq are **inequality symbols**.

Geometrically, $a > b$ means that a is to the right of b (equivalently b is to the left of a) on the real number line. For example, since $6 > 3$, 6 is to the right of 3 on the real number line. Note also that $a > 0$ means that $a - 0$, or simply a , is positive and $a < 0$ means that a is negative.

We are able to compare any two real numbers because of the following important property of the real numbers.

Trichotomy Property

Let a and b be any two real numbers. Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b.$$

Inequalities can be used to describe **intervals** of real numbers, as illustrated in Example 2.

EXAMPLE 2 Interpreting Inequalities

Describe and graph the interval of real numbers for the inequality.

(a) $x < 3$ (b) $-1 < x \leq 4$

SOLUTION

(a) The inequality $x < 3$ describes all real numbers less than 3 (Figure P.3a).

(b) The *double inequality* $-1 < x \leq 4$ represents all real numbers between -1 and 4 , excluding -1 and including 4 (Figure P.3b). **Now try Exercise 5.**

EXAMPLE 3 Writing Inequalities

Write an interval of real numbers using an inequality and draw its graph.

(a) The real numbers between -4 and -0.5

(b) The real numbers greater than or equal to zero

SOLUTION

(a) $-4 < x < -0.5$ (Figure P.3c)




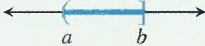
(b) $x \geq 0$ (Figure P.3d)

Now try Exercise 15.

As shown in Example 2, inequalities define *intervals* on the real number line. We often use $[2, 5]$ to describe the *bounded interval* determined by $2 \leq x \leq 5$. This interval is **closed** because it contains its *endpoints* 2 and 5. There are four types of **bounded intervals**.

Bounded Intervals of Real Numbers

Let a and b be real numbers with $a < b$.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$	Half-open	$a \leq x < b$	
$(a, b]$	Half-open	$a < x \leq b$	

The numbers a and b are the **endpoints** of each interval.

INTERVAL NOTATION AT $\pm\infty$


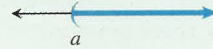


Because $-\infty$ is *not* a real number, we use $(-\infty, 2)$ instead of $[-\infty, 2)$ to describe $x < 2$. Similarly, we use $[-1, \infty)$ instead of $[-1, \infty]$ to describe $x \geq -1$.

The interval of real numbers determined by the inequality $x < 2$ can be described by the *unbounded interval* $(-\infty, 2)$. This interval is **open** because it does *not* contain its endpoint 2.

We use the interval notation $(-\infty, \infty)$ to represent the entire set of real numbers. The symbols $-\infty$ (*negative infinity*) and ∞ (*positive infinity*) allow us to use interval notation for unbounded intervals and are *not* real numbers. There are four types of **unbounded intervals**.

Unbounded Intervals of Real Numbers

Let a and b be real numbers.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, \infty)$	Closed	$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$	Closed	$x \leq b$	
$(-\infty, b)$	Open	$x < b$	

Each of these intervals has exactly one endpoint, namely a or b .

EXAMPLE 4 Converting Between Intervals and Inequalities

Convert interval notation to inequality notation or vice versa. Find the endpoints and state whether the interval is bounded, its type, and graph the interval.

(a) $[-6, 3)$ (b) $(-\infty, -1)$ (c) $-2 \leq x \leq 3$

SOLUTION

(a) The interval $[-6, 3)$ corresponds to $-6 \leq x < 3$ and is bounded and half-open (see Figure P.4a). The endpoints are -6 and 3 .

(b) The interval $(-\infty, -1)$ corresponds to $x < -1$ and is unbounded and open (see Figure P.4b). The only endpoint is -1 .

(c) The inequality $-2 \leq x \leq 3$ corresponds to the closed, bounded interval $[-2, 3]$ (see Figure P.4c). The endpoints are -2 and 3 .

Now try Exercise 29.

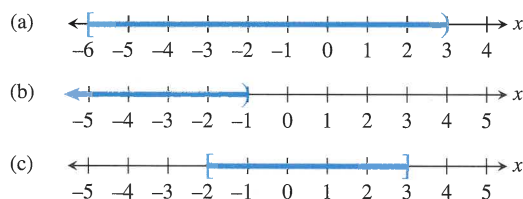


FIGURE P.4 Graphs of the intervals of real numbers in Example 4.

Basic Properties of Algebra

Algebra involves the use of letters and other symbols to represent real numbers. A **variable** is a letter or symbol (for example, x , y , t , θ) that represents an unspecified real number. A **constant** is a letter or symbol (for example, -2 , 0 , $\sqrt{3}$, π) that represents a specific real number. An **algebraic expression** is a combination of variables and constants involving addition, subtraction, multiplication, division, powers, and roots.

We state some of the properties of the arithmetic operations of addition, subtraction, multiplication, and division, represented by the symbols $+$, $-$, \times (or \cdot) and \div (or $/$), respectively. Addition and multiplication are the primary operations. Subtraction and division are defined in terms of addition and multiplication.

Subtraction: $a - b = a + (-b)$

Division: $\frac{a}{b} = a\left(\frac{1}{b}\right), b \neq 0$

In the above definitions, $-b$ is the **additive inverse** or **opposite** of b , and $1/b$ is the **multiplicative inverse** or **reciprocal** of b . Perhaps surprisingly, additive inverses are not always negative numbers. The additive inverse of 5 is the negative number -5 . However, the additive inverse of -3 is the positive number 3 .

SUBTRACTION VS. NEGATIVE NUMBERS

On many calculators, there are two “ $-$ ” keys, one for subtraction and one for negative numbers or opposites. Be sure you know how to use both keys correctly. Misuse can lead to incorrect results.

The following properties hold for real numbers, variables, and algebraic expressions.

Properties of Algebra

Let u , v , and w be real numbers, variables, or algebraic expressions.

1. Commutative property

Addition: $u + v = v + u$

Multiplication: $uv = vu$

2. Associative property

Addition:

$$(u + v) + w = u + (v + w)$$

Multiplication: $(uv)w = u(vw)$

3. Identity property

Addition: $u + 0 = u$

Multiplication: $u \cdot 1 = u$

4. Inverse property

Addition: $u + (-u) = 0$

Multiplication: $u \cdot \frac{1}{u} = 1, u \neq 0$

5. Distributive property

Multiplication over addition:

$$u(v + w) = uv + uw$$

$$(u + v)w = uw + vw$$

Multiplication over subtraction:

$$u(v - w) = uv - uw$$

$$(u - v)w = uw - vw$$

The left-hand sides of the equations for the distributive property show the **factored form** of the algebraic expressions, and the right-hand sides show the **expanded form**.

EXAMPLE 5 Using the Distributive Property

(a) Write the expanded form of $(a + 2)x$.

(b) Write the factored form of $3y - by$.

SOLUTION

(a) $(a + 2)x = ax + 2x$

(b) $3y - by = (3 - b)y$

Now try Exercise 37.

Here are some properties of the additive inverse together with examples that help illustrate their meanings.

Properties of the Additive Inverse

Let u and v be real numbers, variables, or algebraic expressions.

Property

Example

1. $-(-u) = u$

$$-(-3) = 3$$

2. $(-u)v = u(-v) = -(uv)$

$$(-4)3 = 4(-3) = -(4 \cdot 3) = -12$$

3. $(-u)(-v) = uv$

$$(-6)(-7) = 6 \cdot 7 = 42$$

4. $(-1)u = -u$

$$(-1)5 = -5$$

5. $-(u + v) = (-u) + (-v)$

$$-(7 + 9) = (-7) + (-9) = -16$$

Integer Exponents

Exponential notation is used to shorten products of factors that repeat. For example,

$$(-3)(-3)(-3)(-3) = (-3)^4 \quad \text{and} \quad (2x + 1)(2x + 1) = (2x + 1)^2.$$

Exponential Notation

Let a be a real number, variable, or algebraic expression and n a positive integer. Then

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ factors}},$$

where n is the **exponent**, a is the **base**, and a^n is the **n th power of a** , read as “ a to the n th power.”

The two exponential expressions in Example 6 have the same value but have different bases. Be sure you understand the difference.

EXAMPLE 6 Identifying the Base

(a) In $(-3)^5$, the base is -3 .

(b) In -3^5 , the base is 3 .

Now try Exercise 43.

UNDERSTANDING NOTATION

$$(-3)^2 = 9$$

$$-3^2 = -9$$

Be careful!

Here are the basic properties of exponents together with examples that help illustrate their meanings.

Properties of Exponents

Let u and v be real numbers, variables, or algebraic expressions and m and n be integers. All bases are assumed to be nonzero.

Property

$$1. u^m u^n = u^{m+n}$$

$$2. \frac{u^m}{u^n} = u^{m-n}$$

$$3. u^0 = 1$$

$$4. u^{-n} = \frac{1}{u^n}$$

$$5. (uv)^m = u^m v^m$$

$$6. (u^m)^n = u^{mn}$$

$$7. \left(\frac{u}{v}\right)^m = \frac{u^m}{v^m}$$

Example

$$5^3 \cdot 5^4 = 5^{3+4} = 5^7$$

$$\frac{x^9}{x^4} = x^{9-4} = x^5$$

$$8^0 = 1$$

$$y^{-3} = \frac{1}{y^3}$$

$$(2z)^5 = 2^5 z^5 = 32z^5$$

$$(x^2)^3 = x^{2 \cdot 3} = x^6$$

$$\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$$

To simplify an expression involving powers means to rewrite it so that each factor appears only once, all exponents are positive, and exponents and constants are combined as much as possible.

MOVING FACTORS

Be sure you understand how exponent Property 4 permits us to move factors from the numerator to the denominator and vice versa:

$$\frac{v^{-m}}{u^{-n}} = \frac{u^n}{v^m}$$

EXAMPLE 7 Simplifying Expressions Involving Powers

$$(a) (2ab^3)(5a^2b^5) = 10(aa^2)(b^3b^5) = 10a^3b^8$$

$$(b) \frac{u^2v^{-2}}{u^{-1}v^3} = \frac{u^2u^1}{v^2v^3} = \frac{u^3}{v^5}$$

$$(c) \left(\frac{x^2}{2}\right)^{-3} = \frac{(x^2)^{-3}}{2^{-3}} = \frac{x^{-6}}{2^{-3}} = \frac{2^3}{x^6} = \frac{8}{x^6}$$

Now try Exercise 47.



Scientific Notation

Any positive number can be written in **scientific notation**,

$$c \times 10^m, \text{ where } 1 \leq c < 10 \text{ and } m \text{ is an integer.}$$

This notation provides a way to work with very large and very small numbers. For example, the distance between the Earth and the Sun is about 93,000,000 miles. In scientific notation,

$$93,000,000 \text{ mi} = 9.3 \times 10^7 \text{ mi.}$$

The *positive exponent* 7 indicates that moving the decimal point in 9.3 to the right 7 places produces the decimal form of the number.

The mass of an oxygen molecule is about

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 053 \text{ grams.}$$

In scientific notation,

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 053 \text{ g} = 5.3 \times 10^{-23} \text{ g.}$$

The *negative exponent* -23 indicates that moving the decimal point in 5.3 to the left 23 places produces the decimal form of the number.

EXAMPLE 8 Converting to and from Scientific Notation

$$(a) 2.375 \times 10^8 = 237,500,000$$

$$(b) 0.000000349 = 3.49 \times 10^{-7}$$

Now try Exercises 57 and 59.

EXAMPLE 9 Using Scientific Notation

Simplify $\frac{(370,000)(4,500,000,000)}{18,000}$.

SOLUTION

Using Mental Arithmetic

$$\begin{aligned} \frac{(370,000)(4,500,000,000)}{18,000} &= \frac{(3.7 \times 10^5)(4.5 \times 10^9)}{1.8 \times 10^4} \\ &= \frac{(3.7)(4.5)}{1.8} \times 10^{5+9-4} \\ &= 9.25 \times 10^{10} \\ &= 92,500,000,000 \end{aligned}$$

Using a Calculator

Figure P.5 shows two ways to perform the computation. In the first, the numbers are entered in decimal form. In the second, the numbers are entered in scientific notation. The calculator uses “9.25E10” to stand for 9.25×10^{10} .

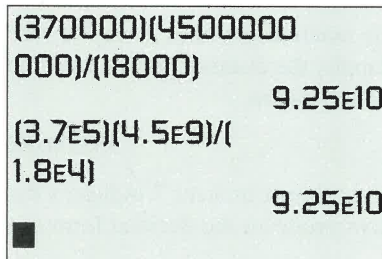


FIGURE P.5 Be sure you understand how your calculator displays scientific notation. (Example 9)

Now try Exercise 63.

QUICK REVIEW P.1

- List the positive integers between -3 and 7 .
- List the integers between -3 and 7 .
- List all negative integers greater than -4 .
- List all positive integers less than 5 .

In Exercises 5 and 6, use a calculator to evaluate the expression. Round the value to two decimal places.

- (a) $4(-3.1)^3 - (-4.2)^5$ (b) $\frac{2(-5.5) - 6}{7.4 - 3.8}$
- (a) $5[3(-1.1)^2 - 4(-0.5)^3]$ (b) $5^{-2} + 2^{-4}$

In Exercises 7 and 8, evaluate the algebraic expression for the given values of the variables.

- $x^3 - 2x + 1$, $x = -2, 1.5$
- $a^2 + ab + b^2$, $a = -3, b = 2$

In Exercises 9 and 10, list the possible remainders.

- When the positive integer n is divided by 7 .
- When the positive integer n is divided by 13 .

SECTION P.1 EXERCISES

In Exercises 1–4, find the decimal form for the rational number. State whether it repeats or terminates.

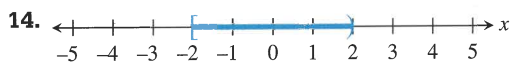
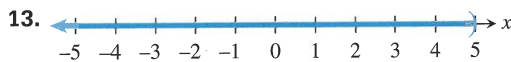
1. $-37/8$ 2. $15/99$
3. $-13/6$ 4. $5/37$

In Exercises 5–10, describe and graph the interval of real numbers.

5. $x \leq 2$ 6. $-2 \leq x < 5$
7. $(-\infty, 7)$ 8. $[-3, 3]$
9. x is negative
10. x is greater than or equal to 2 and less than or equal to 6.

In Exercises 11–16, use an inequality to describe the interval of real numbers.

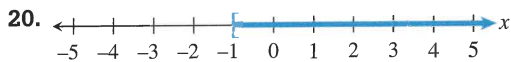
11. $[-1, 1)$ 12. $(-\infty, 4]$



15. x is between -1 and 2 .
16. x is greater than or equal to 5 .

In Exercises 17–22, use interval notation to describe the interval of real numbers.

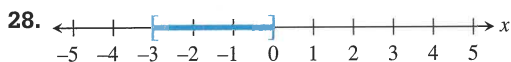
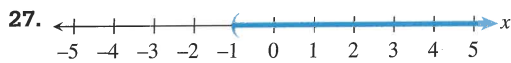
17. $x > -3$ 18. $-7 < x < -2$



21. x is greater than -3 and less than or equal to 4 .
22. x is positive.

In Exercises 23–28, use words to describe the interval of real numbers.

23. $4 < x \leq 9$ 24. $x \geq -1$
25. $[-3, \infty)$ 26. $(-5, 7)$



In Exercises 29–32, convert to inequality notation. Find the endpoints and state whether the interval is bounded or unbounded and its type.

29. $(-3, 4]$ 30. $(-3, -1)$
31. $(-\infty, 5)$ 32. $[-6, \infty)$

In Exercises 33–36, use both inequality and interval notation to describe the set of numbers. State the meaning of any variables you use.

33. **Writing to Learn** Bill is at least 29 years old.
34. **Writing to Learn** No item at Sarah's Variety Store costs more than \$2.00.
35. **Writing to Learn** The price of a gallon of gasoline varies from \$1.099 to \$1.399.
36. **Writing to Learn** Salary raises at the State University of California at Chico will average between 2% and 6.5%.

In Exercises 37–40, use the distributive property to write the factored form or the expanded form of the given expression.

37. $a(x^2 + b)$ 38. $(y - z^3)c$
39. $ax^2 + dx^2$ 40. $a^3z + a^3w$

In Exercises 41 and 42, find the additive inverse of the number.

41. $6 - \pi$ 42. -7

In Exercises 43 and 44, identify the base of the exponential expression.

43. -5^2 44. $(-2)^7$

45. **Group Activity** Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

- (a) $(3x)y = 3(xy)$ (b) $a^2b = ba^2$
(c) $a^2b + (-a^2b) = 0$ (d) $(x + 3)^2 + 0 = (x + 3)^2$
(e) $a(x + y) = ax + ay$

46. **Group Activity** Discuss which algebraic property or properties are illustrated by the equation. Try to reach a consensus.

- (a) $(x + 2) \frac{1}{x + 2} = 1$ (b) $1 \cdot (x + y) = x + y$
(c) $2(x - y) = 2x - 2y$
(d) $2x + (y - z) = 2x + (y + (-z))$
 $= (2x + y) + (-z) =$
 $(2x + y) - z$
(e) $\frac{1}{a}(ab) = \left(\frac{1}{a}\right)b = 1 \cdot b = b$

In Exercises 47–52, simplify the expression. Assume that the variables in the denominators are nonzero.

47. $\frac{x^4y^3}{x^2y^5}$

48. $\frac{(3x^2)^2y^4}{3y^2}$

49. $\left(\frac{4}{x^2}\right)^2$

50. $\left(\frac{2}{xy}\right)^{-3}$

51. $\frac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$

52. $\frac{(4a^3b)\left(\frac{3b^2}{2a^2b^4}\right)}{a^2b^3}$

The data in Table P.1 gives the revenues in thousands of dollars for public elementary and secondary schools for the 2003–04 school year.



Table P.1 Department of Education

Source	Amount (in \$1000)
Federal	36,930,338
State	221,802,107
Local and Intermediate	193,175,805
Total	451,908,251

Source: National Education Association, as reported in *The World Almanac and Book of Facts, 2005*.

In Exercises 53–56, write the amount of revenue in dollars obtained from the source in scientific notation.

53. Federal

54. State

55. Local and Intermediate

56. Total

In Exercises 57 and 58, write the number in scientific notation.

57. The mean distance from Jupiter to the Sun is about 483,900,000 miles.

58. The electric charge, in coulombs, of an electron is about $-0.000\ 000\ 000\ 000\ 000\ 16$.

In Exercises 59–62, write the number in decimal form.

59. 3.33×10^{-8} 60. 6.73×10^{11} 61. The distance that light travels in 1 year (*one light-year*) is about 5.87×10^{12} mi.62. The mass of a neutron is about 1.6747×10^{-24} g.

In Exercises 63 and 64, use scientific notation to simplify.

63. $\frac{(1.35 \times 10^{-7})(2.41 \times 10^8)}{1.25 \times 10^9}$

64. $\frac{(3.7 \times 10^{-7})(4.3 \times 10^6)}{2.5 \times 10^7}$

Explorations

- 65. Investigating Exponents** For positive integers m and n , we can use the definition to show that $a^m a^n = a^{m+n}$.
- (a) Examine the equation $a^m a^n = a^{m+n}$ for $n = 0$ and explain why it is reasonable to define $a^0 = 1$ for $a \neq 0$.
- (b) Examine the equation $a^m a^n = a^{m+n}$ for $n = -m$ and explain why it is reasonable to define $a^{-m} = 1/a^m$ for $a \neq 0$.
- 66. Decimal Forms of Rational Numbers** Here is the third step when we divide 1 by 17. (The first two steps are not shown, because the quotient is 0 in both cases.)

$$\begin{array}{r} 0.05 \\ 17 \overline{)1.00} \\ \underline{85} \\ 15 \end{array}$$

By convention we say that 1 is the first remainder in the long division process, 10 is the second, and 15 is the third remainder.

- (a) Continue this long division process until a remainder is repeated, and complete the following table:

Step	Quotient	Remainder
1	0	1
2	0	10
3	5	15
⋮	⋮	⋮

- (b) Explain why the digits that occur in the quotient between the pair of repeating remainders determine the infinitely repeating portion of the decimal representation. In this case

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

- (c) Explain why this procedure will always determine the infinitely repeating portion of a rational number whose decimal representation does not terminate.

Standardized Test Questions

67. **True or False** The additive inverse of a real number must be negative. Justify your answer.
68. **True or False** The reciprocal of a positive real number must be less than 1. Justify your answer.

In Exercises 69–72, solve these problems without using a calculator.

69. **Multiple Choice** Which of the following inequalities corresponds to the interval $[-2, 1)$?
- (A) $x \leq -2$ (B) $-2 \leq x \leq 1$
(C) $-2 < x < 1$ (D) $-2 < x \leq 1$
(E) $-2 \leq x < 1$
70. **Multiple Choice** What is the value of $(-2)^4$?
- (A) 16 (B) 8
(C) 6 (D) -8
(E) -16
71. **Multiple Choice** What is the base of the exponential expression -7^2 ?
- (A) -7 (B) 7
(C) -2 (D) 2
(E) 1

72. **Multiple Choice** Which of the following is the simplified form of $\frac{x^6}{x^2}$, $x \neq 0$?
- (A) x^{-4} (B) x^2
(C) x^3 (D) x^4
(E) x^8

Extending the Ideas

The **magnitude** of a real number is its distance from the origin.

73. List the whole numbers whose magnitudes are less than 7.
74. List the natural numbers whose magnitudes are less than 7.
75. List the integers whose magnitudes are less than 7.

P.2

Cartesian Coordinate System

What you'll learn about

- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications

... and why

These topics provide the foundation for the material that will be covered in this textbook.

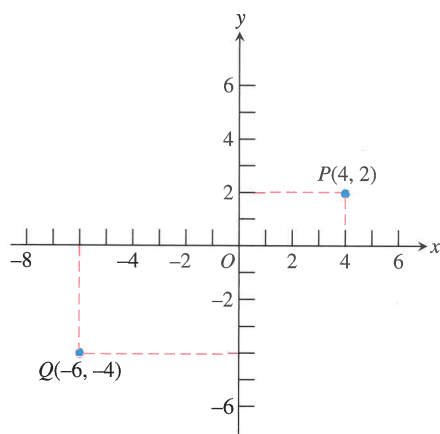


FIGURE P.6 The Cartesian coordinate plane.

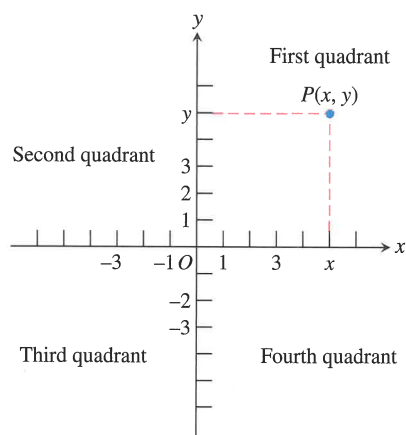


FIGURE P.7 The four quadrants. Points on the x - or y -axis are not in any quadrant.

Cartesian Plane

The points in a plane correspond to ordered pairs of real numbers, just as the points on a line can be associated with individual real numbers. This correspondence creates the **Cartesian plane**, or the **rectangular coordinate system** in the plane.

To construct a rectangular coordinate system, or a Cartesian plane, draw a pair of perpendicular real number lines, one horizontal and the other vertical, with the lines intersecting at their respective 0-points (Figure P.6). The horizontal line is usually the **x -axis** and the vertical line is usually the **y -axis**. The positive direction on the x -axis is to the right, and the positive direction on the y -axis is up. Their point of intersection, O , is the **origin of the Cartesian plane**.

Each point P of the plane is associated with an **ordered pair (x, y)** of real numbers, the **(Cartesian) coordinates of the point**. The **x -coordinate** represents the intersection of the x -axis with the perpendicular from P , and the **y -coordinate** represents the intersection of the y -axis with the perpendicular from P . Figure P.6 shows the points P and Q with coordinates $(4, 2)$ and $(-6, -4)$, respectively. As with real numbers and a number line, we use the ordered pair (a, b) for both the name of the point and its coordinates.

The coordinate axes divide the Cartesian plane into four **quadrants**, as shown in Figure P.7.

EXAMPLE 1 Plotting Data on U.S. Exports to Mexico

The value in billions of dollars of U.S. exports to Mexico from 1996 to 2003 is given in Table P.2. Plot the (year, export value) ordered pairs in a rectangular coordinate system.



Table P.2 U.S. Exports to Mexico

Year	U.S. Exports (billions of dollars)
1996	56.8
1997	71.4
1998	78.8
1999	86.9
2000	111.3
2001	101.3
2002	97.5
2003	97.4

Source: U.S. Census Bureau, *Statistical Abstract of the United States, 2001, 2004–2005*.

SOLUTION

The points are plotted in Figure P.8 on page 15.

Now try Exercise 31.

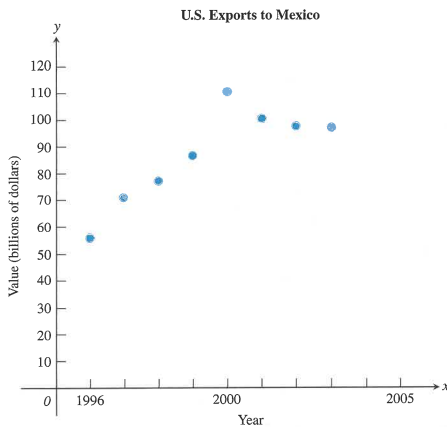


FIGURE P.8 The graph for Example 1.

A **scatter plot** is a plotting of the (x, y) data pairs on a Cartesian plane. Figure P.8 shows a scatter plot of the data from Table P.2.

Absolute Value of a Real Number

The *absolute value of a real number* suggests its **magnitude** (size). For example, the absolute value of 3 is 3 and the absolute value of -5 is 5.

DEFINITION Absolute Value of a Real Number

The **absolute value of a real number a** is

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0. \\ 0, & \text{if } a = 0 \end{cases}$$

EXAMPLE 2 Using the Definition of Absolute Value

Evaluate:

(a) $|-4|$ (b) $|\pi - 6|$

SOLUTION

(a) Because $-4 < 0$, $|-4| = -(-4) = 4$.

(b) Because $\pi \approx 3.14$, $\pi - 6$ is negative, so $\pi - 6 < 0$. Thus,
 $|\pi - 6| = -(\pi - 6) = 6 - \pi \approx 2.858$.

Now try Exercise 9.

Here is a summary of some important properties of absolute value.

Properties of Absolute Value

Let a and b be real numbers.

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $b \neq 0$

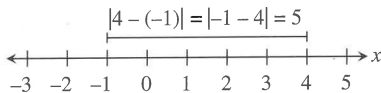


FIGURE P.9 Finding the distance between -1 and 4 .

ABSOLUTE VALUE AND DISTANCE

If we let $b = 0$ in the distance formula we see that the distance between a and 0 is $|a|$. Thus, the absolute value of a number is its distance from zero.

Distance Formulas

The *distance* between -1 and 4 on the number line is 5 (see Figure P.9). This distance may be found by subtracting the smaller number from the larger: $4 - (-1) = 5$. If we use absolute value, the order of subtraction does not matter: $|4 - (-1)| = |-1 - 4| = 5$.

Distance Formula (Number Line)

Let a and b be real numbers. The **distance between a and b** is

$$|a - b|.$$

Note that $|a - b| = |b - a|$.

To find the *distance* between two points that lie on the same horizontal or vertical line in the Cartesian plane, we use the distance formula for points on a number line. For example, the distance between points x_1 and x_2 on the x -axis is $|x_1 - x_2| = |x_2 - x_1|$ and the distance between the points y_1 and y_2 on the y -axis is $|y_1 - y_2| = |y_2 - y_1|$.

To find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ that do not lie on the same horizontal or vertical line we form the right triangle determined by P , Q , and $R(x_2, y_1)$ (Figure P.10).

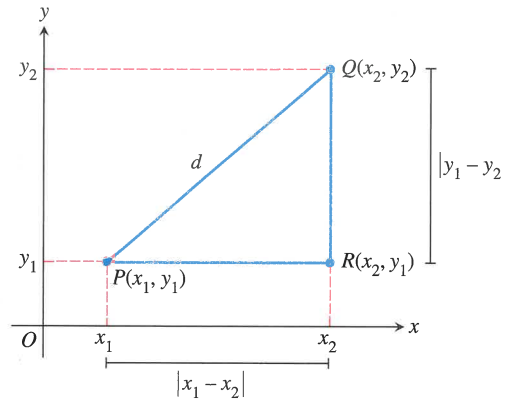


FIGURE P.10 Forming a right triangle with hypotenuse \overline{PQ} .

The distance from P to R is $|x_1 - x_2|$, and the distance from R to Q is $|y_1 - y_2|$. By the **Pythagorean Theorem** (see Figure P.11), the distance d between P and Q is

$$d = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}.$$

Because $|x_1 - x_2|^2 = (x_1 - x_2)^2$ and $|y_1 - y_2|^2 = (y_1 - y_2)^2$, we obtain the following formula.

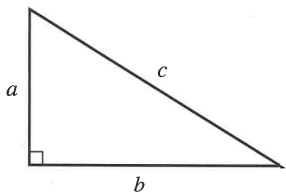


FIGURE P.11 The Pythagorean theorem: $c^2 = a^2 + b^2$.

Distance Formula (Coordinate Plane)

The distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

EXAMPLE 3 Finding the Distance Between Two Points

Find the distance d between the points $(1, 5)$ and $(6, 2)$.

SOLUTION

$$\begin{aligned} d &= \sqrt{(1 - 6)^2 + (5 - 2)^2} && \text{The distance formula} \\ &= \sqrt{(-5)^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \approx 5.83 && \text{Using a calculator} \end{aligned}$$

Now try Exercise 11.

Midpoint Formulas

When the endpoints of a segment in a number line are known, we take the average of their coordinates to find the midpoint of the segment.

Midpoint Formula (Number Line)

The midpoint of the line segment with endpoints a and b is

$$\frac{a + b}{2}$$

EXAMPLE 4 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints -9 and 3 on a number line is

$$\frac{(-9) + 3}{2} = \frac{-6}{2} = -3.$$

See Figure P.12.

Now try Exercise 23.

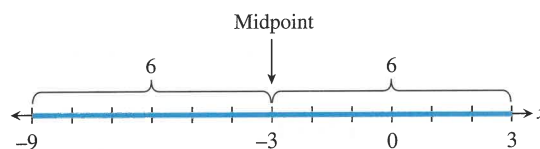


FIGURE P.12 Notice that the distance from the midpoint, -3 , to 3 or to -9 is 6 . (Example 4)

Just as with number lines, the midpoint of a line segment in the coordinate plane is determined by its endpoints. Each coordinate of the midpoint is the average of the corresponding coordinates of its endpoints.

Midpoint Formula (Coordinate Plane)

The midpoint of the line segment with endpoints (a, b) and (c, d) is

$$\left(\frac{a + c}{2}, \frac{b + d}{2} \right).$$

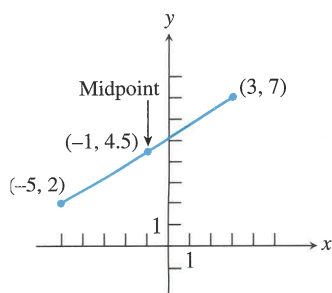


FIGURE P.13 (Example 5.)

EXAMPLE 5 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints $(-5, 2)$ and $(3, 7)$ is

$$(x, y) = \left(\frac{-5 + 3}{2}, \frac{2 + 7}{2} \right) = (-1, 4.5).$$

See Figure P.13.

Now try Exercise 25.

Equations of Circles

A **circle** is the set of points in a plane at a fixed distance (**radius**) from a fixed point (**center**). Figure P.14 shows the circle with center (h, k) and radius r . If (x, y) is any point on the circle, the distance formula gives

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

Squaring both sides, we obtain the following equation for a circle.

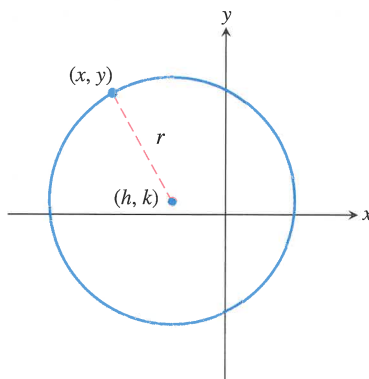


FIGURE P.14 The circle with center (h, k) and radius r .

DEFINITION Standard Form Equation of a Circle

The **standard form equation of a circle** with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

EXAMPLE 6 Finding Standard Form Equations of Circles

Find the standard form equation of the circle.

- (a) Center $(-4, 1)$, radius 8 (b) Center $(0, 0)$, radius 5

SOLUTION

(a) $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation
 $(x - (-4))^2 + (y - 1)^2 = 8^2$ Substitute $h = -4, k = 1, r = 8$.
 $(x + 4)^2 + (y - 1)^2 = 64$

(b) $(x - h)^2 + (y - k)^2 = r^2$ Standard form equation
 $(x - 0)^2 + (y - 0)^2 = 5^2$ Substitute $h = 0, k = 0, r = 5$.
 $x^2 + y^2 = 25$

Now try Exercise 41.

Applications

EXAMPLE 7 Using an Inequality to Express Distance

We can state that “the distance between x and -3 is less than 9” using the inequality

$$|x - (-3)| < 9 \quad \text{or} \quad |x + 3| < 9.$$

Now try Exercise 51.

The converse of the Pythagorean theorem is true. That is, if the sum of squares of the lengths of the two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

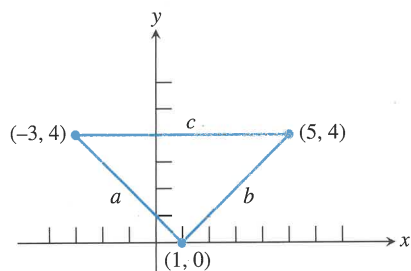


FIGURE P.15 The triangle in Example 8.

EXAMPLE 8 Verifying Right Triangles

Use the converse of the Pythagorean theorem and the distance formula to show that the points $(-3, 4)$, $(1, 0)$, and $(5, 4)$ determine a right triangle.

SOLUTION The three points are plotted in Figure P.15. We need to show that the lengths of the sides of the triangle satisfy the Pythagorean relationship $a^2 + b^2 = c^2$. Applying the distance formula we find that

$$a = \sqrt{(-3 - 1)^2 + (4 - 0)^2} = \sqrt{32},$$

$$b = \sqrt{(1 - 5)^2 + (0 - 4)^2} = \sqrt{32},$$

$$c = \sqrt{(-3 - 5)^2 + (4 - 4)^2} = \sqrt{64}.$$

The triangle is a right triangle because

$$a^2 + b^2 = (\sqrt{32})^2 + (\sqrt{32})^2 = 32 + 32 = 64 = c^2.$$

Now try Exercise 39.

Properties of geometric figures can sometimes be confirmed using analytic methods such as the midpoint formulas.

EXAMPLE 9 Using the Midpoint Formula

It is a fact from geometry that the diagonals of a parallelogram bisect each other. Prove this with a midpoint formula.

SOLUTION We can position a parallelogram in the rectangular coordinate plane as shown in Figure P.16. Applying the midpoint formula for the coordinate plane to segments OB and AC , we find that

$$\text{midpoint of segment } OB = \left(\frac{0 + a + c}{2}, \frac{0 + b}{2} \right) = \left(\frac{a + c}{2}, \frac{b}{2} \right),$$

$$\text{midpoint of segment } AC = \left(\frac{a + c}{2}, \frac{b + 0}{2} \right) = \left(\frac{a + c}{2}, \frac{b}{2} \right).$$

The midpoints of segments OB and AC are the same, so the diagonals of the parallelogram $OACB$ meet at their midpoints and thus bisect each other.

Now try Exercise 37.

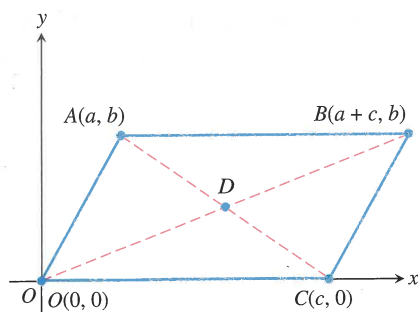


FIGURE P.16 The coordinates of B must be $(a + c, b)$ in order for CB to be parallel to OA . (Example 9)

QUICK REVIEW P.2

In Exercises 1 and 2, plot the two numbers on a number line. Then find the distance between them.

1. $\sqrt{7}, \sqrt{2}$ 2. $-\frac{5}{3}, -\frac{9}{5}$

In Exercises 3 and 4, plot the real numbers on a number line.

3. $-3, 4, 2.5, 0, -1.5$ 4. $-\frac{5}{2}, -\frac{1}{2}, \frac{2}{3}, 0, -1$

In Exercises 5 and 6, plot the points.

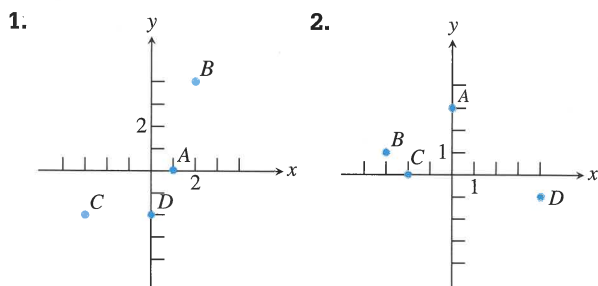
5. $A(3, 5), B(-2, 4), C(3, 0), D(0, -3)$
 6. $A(-3, -5), B(2, -4), C(0, 5), D(-4, 0)$

In Exercises 7–10, use a calculator to evaluate the expression. Round your answer to two decimal places.

7. $\frac{-17 + 28}{2}$ 8. $\sqrt{13^2 + 17^2}$
 9. $\sqrt{6^2 + 8^2}$ 10. $\sqrt{(17 - 3)^2 + (-4 - 8)^2}$

SECTION P.2 EXERCISES

In Exercises 1 and 2, estimate the coordinates of the points.



In Exercises 3 and 4, name the quadrants containing the points.

3. (a) $(2, 4)$ (b) $(0, 3)$ (c) $(-2, 3)$ (d) $(-1, -4)$

4. (a) $(\frac{1}{2}, \frac{3}{2})$ (b) $(-2, 0)$ (c) $(-1, -2)$ (d) $(-\frac{3}{2}, -\frac{7}{3})$

In Exercises 5–8, evaluate the expression.

5. $3 + |-3|$ 6. $2 - |-2|$

7. $|(-2)3|$ 8. $\frac{-2}{|-2|}$

In Exercises 9 and 10, rewrite the expression without using absolute value symbols.

9. $|\pi - 4|$ 10. $|\sqrt{5} - 5/2|$

In Exercises 11–18, find the distance between the points.

11. $-9.3, 10.6$

12. $-5, -17$

13. $(-3, -1), (5, -1)$

14. $(-\sqrt{4}, -3), (\sqrt{1}, 1)$

15. $(0, 0), (3, 4)$

16. $(-1, 2), (2, -3)$

17. $(-2, 0), (5, 0)$

18. $(0, -8), (0, -1)$

In Exercises 19–22, find the area and perimeter of the figure determined by the points.

19. $(-5, 3), (0, -1), (4, 4)$

20. $(-2, -2), (-2, 2), (2, 2), (2, -2)$

21. $(-3, -1), (-1, 3), (7, 3), (5, -1)$

22. $(-2, 1), (-2, 6), (4, 6), (4, 1)$

In Exercises 23–28, find the midpoint of the line segment with the given endpoints.

23. $-9.3, 10.6$

24. $-5, -17$

25. $(-1, 3), (5, 9)$

26. $(3, \sqrt{2}), (6, 2)$

27. $(-7/3, 3/4), (5/3, -9/4)$

28. $(5, -2), (-1, -4)$

In Exercises 29–34, draw a scatter plot of the data given in the table.

29. **U.S. Aluminum Imports** The total value y in billions of dollars of aluminum imported by the United States each year from 1997 to 2003 is given in the table. (Source: *U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.*)

x	1997	1998	1999	2000	2001	2002	2003
y	5.6	6.0	6.3	6.9	6.4	6.6	7.2

30. **U.S. Aluminum Exports** The total value y in billions of dollars of aluminum exported by the United States each year from 1997 to 2003 is given in the table. (Source: *U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.*)

x	1997	1998	1999	2000	2001	2002	2003
y	3.8	3.6	3.6	3.8	3.3	2.9	2.9

- 31. U.S. Imports from Mexico** The total in billions of dollars of U.S. imports from Mexico from 1996 to 2003 is given in Table P.3.



Table P.3 U.S. Imports from Mexico

Year	U.S. Imports (billions of dollars)
1996	74.3
1997	85.9
1998	94.6
1999	109.7
2000	135.9
2001	131.3
2002	134.6
2003	138.1

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.

- 32. U.S. Agricultural Exports** The total in billions of dollars of U.S. agricultural exports from 1996 to 2003 is given in Table P.4.



Table P.4 U.S. Agricultural Exports

Year	U.S. Agricultural Exports (billions of dollars)
1996	60.6
1997	57.1
1998	52.0
1999	48.2
2000	53.0
2001	55.2
2002	54.8
2003	61.5

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2004–2005.

- 33. U.S. Agricultural Trade Surplus** The total in billions of dollars of U.S. agricultural trade surplus from 1996 to 2003 is given in Table P.5.



Table P.5 U.S. Agricultural Trade Surplus

Year	U.S. Agricultural Trade Surplus (billions of dollars)
1996	28.1
1997	21.9
1998	16.3
1999	11.5
2000	13.8
2001	15.7
2002	12.8
2003	8.3

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2004–2005.

- 34. U.S. Exports to Armenia** The total in millions of dollars of U.S. exports to Armenia from 1996 to 2003 is given in Table P.6.

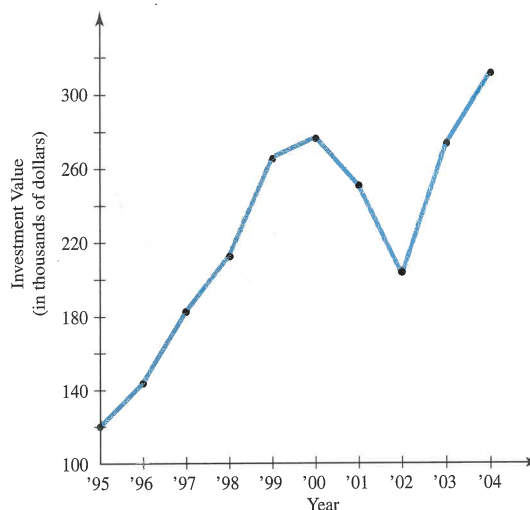


Table P.6 U.S. Exports to Armenia

Year	U.S. Exports (millions of dollars)
1996	57.4
1997	62.1
1998	51.4
1999	51.2
2000	55.6
2001	49.9
2002	111.8
2003	102.8

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.

In Exercises 35 and 36, use the graph of the investment value of a \$10,000 investment made in 1978 in Fundamental Investors™ of the American Funds™. The value as of January is shown for a few recent years in the graph below. (Source: Annual report of Fundamental Investors for the year ending December 31, 2004.)



- 35. Reading from Graphs** Use the graph to estimate the value of the investment as of
 (a) January 1997 and (b) January 2000.
- 36. Percent Increase** Estimate the percent increase in the value of the \$10,000 investment from
 (a) January 1996 to January 1997.
 (b) January 2000 to January 2001.
 (c) January 1995 to January 2004.

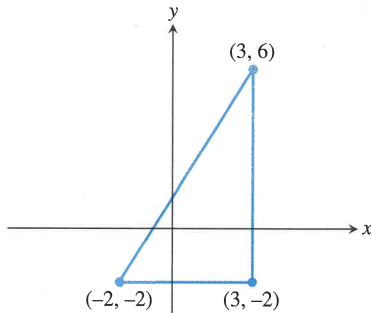
37. Prove that the figure determined by the points is an isosceles triangle: (1, 3), (4, 7), (8, 4)

38. **Group Activity** Prove that the diagonals of the figure determined by the points bisect each other.

(a) Square (-7, -1), (-2, 4), (3, -1), (-2, -6)

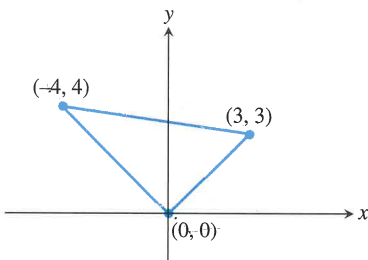
(b) Parallelogram (-2, -3), (0, 1), (6, 7), (4, 3)

39. (a) Find the lengths of the sides of the triangle in the figure.



(b) **Writing to Learn** Show that the triangle is a right triangle.

40. (a) Find the lengths of the sides of the triangle in the figure.



(b) **Writing to Learn** Show that the triangle is a right triangle.

In Exercises 41–44, find the standard form equation for the circle.

41. Center (1, 2), radius 5

42. Center (-3, 2), radius 1

43. Center (-1, -4), radius 3

44. Center (0, 0), radius $\sqrt{3}$

In Exercises 45–48, find the center and radius of the circle.

45. $(x - 3)^2 + (y - 1)^2 = 36$

46. $(x + 4)^2 + (y - 2)^2 = 121$

47. $x^2 + y^2 = 5$

48. $(x - 2)^2 + (y + 6)^2 = 25$

In Exercises 49–52, write the statement using absolute value notation.

49. The distance between x and 4 is 3.

50. The distance between y and -2 is greater than or equal to 4.

51. The distance between x and c is less than d units.

52. y is more than d units from c .

53. **Determining a Line Segment with Given Midpoint**

Let (4, 4) be the midpoint of the line segment determined by the points (1, 2) and (a, b) . Determine a and b .

54. **Writing to Learn Isosceles but Not Equilateral Triangle**

Prove that the triangle determined by the points (3, 0), (-1, 2), and (5, 4) is isosceles but not equilateral.

55. **Writing to Learn Equidistant Point from Vertices of a Right Triangle**

Prove that the midpoint of the hypotenuse of the right triangle with vertices (0, 0), (5, 0), and (0, 7) is equidistant from the three vertices.

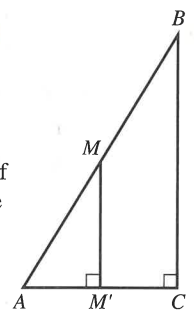
56. **Writing to Learn** Describe the set of real numbers that satisfy $|x - 2| < 3$.

57. **Writing to Learn** Describe the set of real numbers that satisfy $|x + 3| \geq 5$.

Standardized Test Questions

58. **True or False** If a is a real number, then $|a| \geq 0$. Justify your answer.

59. **True or False** Consider the right triangle ABC shown at the right. If M is the midpoint of the segment AB , then M' is the midpoint of the segment AC . Justify your answer.



In Exercises 60–63, solve these problems without using a calculator.

60. **Multiple Choice** Which of the following is equal to

$$|1 - \sqrt{3}|?$$

(A) $1 - \sqrt{3}$

(B) $\sqrt{3} - 1$

(C) $(1 - \sqrt{3})^2$

(D) $\sqrt{2}$

(E) $\sqrt{1/3}$

61. **Multiple Choice** Which of the following is the midpoint of the line segment with endpoints -3 and 2 ?

(A) $5/2$

(B) 1

(C) $-1/2$

(D) -1

(E) $-5/2$

62. **Multiple Choice** Which of the following is the center of the circle $(x - 3)^2 + (y + 4)^2 = 2$?
- (A) (3, -4) (B) (-3, 4)
(C) (4, -3) (D) (-4, 3)
(E) (3/2, -2)
63. **Multiple Choice** Which of the following points is in the third quadrant?
- (A) (0, -3) (B) (-1, 0)
(C) (2, -1) (D) (-1, 2)
(E) (-2, -3)

Explorations

64. Dividing a Line Segment Into Thirds

- (a) Find the coordinates of the points one-third and two-thirds of the way from $a = 2$ to $b = 8$ on a number line.
- (b) Repeat (a) for $a = -3$ and $b = 7$.
- (c) Find the coordinates of the points one-third and two-thirds of the way from a to b on a number line.
- (d) Find the coordinates of the points one-third and two-thirds of the way from the point (1, 2) to the point (7, 11) in the coordinate plane.
- (e) Find the coordinates of the points one-third and two-thirds of the way from the point (a, b) to the point (c, d) in the coordinate plane.

Extending the Ideas

65. **Writing to Learn Equidistant Point from Vertices of a Right Triangle** Prove that the midpoint of the hypotenuse of any right triangle is equidistant from the three vertices.

66. **Comparing Areas** Consider the four points $A(0, 0)$, $B(0, a)$, $C(a, a)$, and $D(a, 0)$. Let P be the midpoint of the line segment CD and Q the point one-fourth of the way from A to D on segment AD .
- (a) Find the area of triangle BPQ .
- (b) Compare the area of triangle BPQ with the area of square $ABCD$.

In Exercises 67–69, let $P(a, b)$ be a point in the first quadrant.

67. Find the coordinates of the point Q in the fourth quadrant so that PQ is perpendicular to the x -axis.
68. Find the coordinates of the point Q in the second quadrant so that PQ is perpendicular to the y -axis.
69. Find the coordinates of the point Q in the third quadrant so that the origin is the midpoint of the segment PQ .
70. **Writing to Learn** Prove that the distance formula for the number line is a special case of the distance formula for the Cartesian plane.

P.3

Linear Equations and Inequalities

What you'll learn about

- Equations
- Solving Equations
- Linear Equations in One Variable
- Linear Inequalities in One Variable

... and why

These topics provide the foundation for algebraic techniques needed throughout this textbook.

Equations

An **equation** is a statement of equality between two expressions. Here are some properties of equality that we use to solve equations algebraically.

Properties of Equality

Let u , v , w , and z be real numbers, variables, or algebraic expressions.

- | | |
|--------------------------|---|
| 1. Reflexive | $u = u$ |
| 2. Symmetric | If $u = v$, then $v = u$. |
| 3. Transitive | If $u = v$, and $v = w$, then $u = w$. |
| 4. Addition | If $u = v$ and $w = z$, then $u + w = v + z$. |
| 5. Multiplication | If $u = v$ and $w = z$, then $uw = vz$. |

Solving Equations

A **solution of an equation in x** is a value of x for which the equation is true. To **solve an equation in x** means to find all values of x for which the equation is true, that is, to find all solutions of the equation.

EXAMPLE 1 Confirming a Solution

Prove that $x = -2$ is a solution of the equation $x^3 - x + 6 = 0$.

SOLUTION

$$\begin{aligned} (-2)^3 - (-2) + 6 &\stackrel{?}{=} 0 \\ -8 + 2 + 6 &\stackrel{?}{=} 0 \\ 0 &= 0 \end{aligned}$$

Now try Exercise 1.

Linear Equations in One Variable

The most basic equation in algebra is a *linear equation*.

DEFINITION Linear Equation in x

A **linear equation in x** is one that can be written in the form

$$ax + b = 0,$$

where a and b are real numbers with $a \neq 0$.

The equation $2z - 4 = 0$ is linear in the variable z . The equation $3u^2 - 12 = 0$ is *not* linear in the variable u . A linear equation in one variable has exactly one solution. We

solve such an equation by transforming it into an *equivalent equation* whose solution is obvious. Two or more equations are **equivalent** if they have the same solutions. For example, the equations $2z - 4 = 0$, $2z = 4$, and $z = 2$ are all equivalent. Here are operations that produce equivalent equations.

Operations for Equivalent Equations

An equivalent equation is obtained if one or more of the following operations are performed.

Operation	Given Equation	Equivalent Equation
1. Combine like terms, reduce fractions, and remove grouping symbols	$2x + x = \frac{3}{9}$	$3x = \frac{1}{3}$
2. Perform the same operation on both sides.		
(a) Add (-3) .	$x + 3 = 7$	$x = 4$
(b) Subtract $(2x)$.	$5x = 2x + 4$	$3x = 4$
(c) Multiply by a nonzero constant $(1/3)$.	$3x = 12$	$x = 4$
(d) Divide by a nonzero constant (3) .	$3x = 12$	$x = 4$

The next two examples illustrate how to use equivalent equations to solve linear equations.

EXAMPLE 2 Solving a Linear Equation

Solve $2(2x - 3) + 3(x + 1) = 5x + 2$. Support the result with a calculator.

SOLUTION

$$2(2x - 3) + 3(x + 1) = 5x + 2$$

$$4x - 6 + 3x + 3 = 5x + 2 \quad \text{Distributive property}$$

$$7x - 3 = 5x + 2 \quad \text{Combine like terms.}$$

$$2x = 5 \quad \text{Add 3, and subtract } 5x.$$

$$x = 2.5 \quad \text{Divide by 2.}$$

To support our algebraic work we can use a calculator to evaluate the original equation for $x = 2.5$. Figure P.17 shows that each side of the original equation is equal to 14.5 if $x = 2.5$.

Now try Exercise 23.

$2.5 \rightarrow X$	2.5
$2(2X-3)+3(X+1)$	14.5
$5X+2$	14.5

FIGURE P.17 The top line stores the number 2.5 into the variable x . (Example 2)

If an equation involves fractions, find the least common denominator (LCD) of the fractions and multiply both sides by the LCD. This is sometimes referred to as *clearing the equation of fractions*. Example 3 illustrates.

INTEGERS AND FRACTIONS

Notice in Example 2 that $2 = \frac{2}{1}$.

EXAMPLE 3 Solving a Linear Equation Involving Fractions

Solve

$$\frac{5y - 2}{8} = 2 + \frac{y}{4}$$

SOLUTION The denominators are 8, 1, and 4. The LCD of the fractions is 8. (See Appendix A.3 if necessary.)

$$\begin{aligned} \frac{5y - 2}{8} &= 2 + \frac{y}{4} \\ 8\left(\frac{5y - 2}{8}\right) &= 8\left(2 + \frac{y}{4}\right) && \text{Multiply by the LCD 8.} \\ 8 \cdot \frac{5y - 2}{8} &= 8 \cdot 2 + 8 \cdot \frac{y}{4} && \text{Distributive property} \\ 5y - 2 &= 16 + 2y && \text{Simplify.} \\ 5y &= 18 + 2y && \text{Add 2.} \\ 3y &= 18 && \text{Subtract 2y.} \\ y &= 6 && \text{Divide by 3.} \end{aligned}$$

We leave it to you to check the solution either using paper and pencil or a calculator.

Now try Exercise 25.

Linear Inequalities in One Variable

We used inequalities to describe order on the number line in Section P.1. For example, if x is to the left of 2 on the number line, or if x is any real number less than 2, we write $x < 2$. The most basic inequality in algebra is a *linear inequality*.

DEFINITION Linear Inequality in x

A **linear inequality in x** is one that can be written in the form

$$ax + b < 0, \quad ax + b \leq 0, \quad ax + b > 0, \quad \text{or} \quad ax + b \geq 0,$$

where a and b are real numbers with $a \neq 0$.

To **solve an inequality in x** means to find all values of x for which the inequality is true. A **solution of an inequality in x** is a value of x for which the inequality is true. The set of all solutions of an inequality is the **solution set** of the inequality. We **solve an inequality** by finding its solution set. Here is a list of properties we use to solve inequalities.

DIRECTION OF AN INEQUALITY

Multiplying (or dividing) an inequality by a positive number preserves the direction of the inequality. Multiplying (or dividing) an inequality by a negative number reverses the direction.

Properties of Inequalities

Let u , v , w , and z be real numbers, variables, or algebraic expressions, and c a real number.

- 1. Transitive** If $u < v$ and $v < w$, then $u < w$.
- 2. Addition** If $u < v$, then $u + w < v + w$.
If $u < v$ and $w < z$, then $u + w < v + z$.
- 3. Multiplication** If $u < v$ and $c > 0$, then $uc < vc$.
If $u < v$ and $c < 0$, then $uc > vc$.

The above properties are true if $<$ is replaced by \leq . There are similar properties for $>$ and \geq .

The set of solutions of a linear inequality in one variable form an interval of real numbers. Just as with linear equations, we solve a linear inequality by transforming it into an *equivalent inequality* whose solutions are obvious. Two or more inequalities are **equivalent** if they have the same set of solutions. The properties of inequalities listed above describe operations that transform an inequality into an equivalent one.

EXAMPLE 4 Solving a Linear Inequality

Solve $3(x - 1) + 2 \leq 5x + 6$.

SOLUTION

$$\begin{aligned}
 3(x - 1) + 2 &\leq 5x + 6 \\
 3x - 3 + 2 &\leq 5x + 6 && \text{Distributive property} \\
 3x - 1 &\leq 5x + 6 && \text{Simplify.} \\
 3x &\leq 5x + 7 && \text{Add 1.} \\
 -2x &\leq 7 && \text{Subtract } 5x. \\
 \left(-\frac{1}{2}\right) \cdot -2x &\geq \left(-\frac{1}{2}\right) \cdot 7 && \text{Multiply by } -1/2. \text{ (The inequality reverses.)} \\
 x &\geq -3.5
 \end{aligned}$$

The solution set of the inequality is the set of all real numbers greater than or equal to -3.5 . In interval notation, the solution set is $[-3.5, \infty)$.

Now try Exercise 41.

Because the solution set of a linear inequality is an interval of real numbers, we can display the solution set with a number line graph as illustrated in Example 5.

EXAMPLE 5 Solving a Linear Inequality Involving Fractions

Solve the inequality and graph its solution set.

$$\frac{x}{3} + \frac{1}{2} > \frac{x}{4} + \frac{1}{3}$$

continued

SOLUTION The LCD of the fractions is 12.

$$\begin{aligned} \frac{x}{3} + \frac{1}{2} &> \frac{x}{4} + \frac{1}{3} && \text{The original inequality} \\ 12 \cdot \left(\frac{x}{3} + \frac{1}{2} \right) &> 12 \cdot \left(\frac{x}{4} + \frac{1}{3} \right) && \text{Multiply by the LCD 12.} \\ 4x + 6 &> 3x + 4 && \text{Simplify.} \\ x + 6 &> 4 && \text{Subtract 3x.} \\ x &> -2 && \text{Subtract 6.} \end{aligned}$$

The solution set is the interval $(-2, \infty)$. Its graph is shown in Figure P.18.

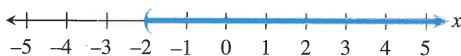


FIGURE P.18 The graph of the solution set of the inequality in Example 5.

Now try Exercise 43.

Sometimes two inequalities are combined in a **double inequality**, whose solution set is a double inequality with x isolated as the middle term. Example 6 illustrates.

EXAMPLE 6 Solving a Double Inequality

Solve the inequality and graph its solution set.

$$-3 < \frac{2x + 5}{3} \leq 5$$

SOLUTION

$$\begin{aligned} -3 < \frac{2x + 5}{3} &\leq 5 \\ -9 < 2x + 5 &\leq 15 && \text{Multiply by 3.} \\ -14 < 2x &\leq 10 && \text{Subtract 5.} \\ -7 < x &\leq 5 && \text{Divide by 2.} \end{aligned}$$

The solution set is the set of all real numbers greater than -7 and less than or equal to 5 . In interval notation, the solution is set $(-7, 5]$. Its graph is shown in Figure P.19.

Now try Exercise 47.

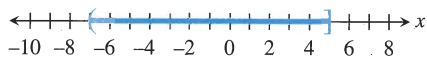


FIGURE P.19 The graph of the solution set of the double inequality in Example 6.

QUICK REVIEW P.3

In Exercises 1 and 2, simplify the expression by combining like terms.

- $2x + 5x + 7 + y - 3x + 4y + 2$
- $4 + 2x - 3z + 5y - x + 2y - z - 2$

In Exercises 3 and 4, use the distributive property to expand the products. Simplify the resulting expression by combining like terms.

- $3(2x - y) + 4(y - x) + x + y$
- $5(2x + y - 1) + 4(y - 3x + 2) + 1$

In Exercises 5–10, use the LCD to combine the fractions. Simplify the resulting fraction.

- $\frac{2}{y} + \frac{3}{y}$
- $\frac{1}{y-1} + \frac{3}{y-2}$
- $2 + \frac{1}{x}$
- $\frac{1}{x} + \frac{1}{y} - x$
- $\frac{x+4}{2} + \frac{3x-1}{5}$
- $\frac{x}{3} + \frac{x}{4}$

SECTION P.3 EXERCISES

In Exercises 1–4, find which values of x are solutions of the equation.

1. $2x^2 + 5x = 3$

(a) $x = -3$ (b) $x = -\frac{1}{2}$ (c) $x = \frac{1}{2}$

2. $\frac{x}{2} + \frac{1}{6} = \frac{x}{3}$

(a) $x = -1$ (b) $x = 0$ (c) $x = 1$

3. $\sqrt{1 - x^2} + 2 = 3$

(a) $x = -2$ (b) $x = 0$ (c) $x = 2$

4. $(x - 2)^{1/3} = 2$

(a) $x = -6$ (b) $x = 8$ (c) $x = 10$

In Exercises 5–10, determine whether the equation is linear in x .

5. $5 - 3x = 0$

6. $5 = 10/2$

7. $x + 3 = x - 5$

8. $x - 3 = x^2$

9. $2\sqrt{x} + 5 = 10$

10. $x + \frac{1}{x} = 1$

In Exercises 11–24, solve the equation.

11. $3x = 24$

12. $4x = -16$

13. $3t - 4 = 8$

14. $2t - 9 = 3$

15. $2x - 3 = 4x - 5$

16. $4 - 2x = 3x - 6$

17. $4 - 3y = 2(y + 4)$

18. $4(y - 2) = 5y$

19. $\frac{1}{2}x = \frac{7}{8}$

20. $\frac{2}{3}x = \frac{4}{5}$

21. $\frac{1}{2}x + \frac{1}{3} = 1$

22. $\frac{1}{3}x + \frac{1}{4} = 1$

23. $2(3 - 4z) - 5(2z + 3) = z - 17$

24. $3(5z - 3) - 4(2z + 1) = 5z - 2$

In Exercises 25–28, solve the equation. Support your answer with a calculator.

25. $\frac{2x - 3}{4} + 5 = 3x$

26. $2x - 4 = \frac{4x - 5}{3}$

27. $\frac{t + 5}{8} - \frac{t - 2}{2} = \frac{1}{3}$

28. $\frac{t - 1}{3} + \frac{t + 5}{4} = \frac{1}{2}$

29. **Writing to Learn** Write a statement about solutions of equations suggested by the computations in the figure.

(a) $-2 \rightarrow X$

$2X^2 + X - 6$	-2
	0

(b) $3/2 \rightarrow X$

$2X^2 + X - 6$	1.5
	0

30. **Writing to Learn** Write a statement about solutions of equations suggested by the computations in the figure.

(a) $2 \rightarrow X$

$7X + 5$	2
$4X - 7$	19
	1

(b) $-4 \rightarrow X$

$7X + 5$	-4
$4X - 7$	-23
	-23

In Exercises 31–34, find which values of x are solutions of the inequality.

31. $2x - 3 < 7$

(a) $x = 0$ (b) $x = 5$ (c) $x = 6$

32. $3x - 4 \geq 5$

(a) $x = 0$ (b) $x = 3$ (c) $x = 4$

33. $-1 < 4x - 1 \leq 11$

(a) $x = 0$ (b) $x = 2$ (c) $x = 3$

34. $-3 \leq 1 - 2x \leq 3$

(a) $x = -1$ (b) $x = 0$ (c) $x = 2$

In Exercises 35–42, solve the inequality, and draw a number line graph of the solution set.

35. $x - 4 < 2$

36. $x + 3 > 5$

37. $2x - 1 \leq 4x + 3$

38. $3x - 1 \geq 6x + 8$

39. $2 \leq x + 6 < 9$

40. $-1 \leq 3x - 2 < 7$

41. $2(5 - 3x) + 3(2x - 1) \leq 2x + 1$

42. $4(1 - x) + 5(1 + x) > 3x - 1$

In Exercises 43–54, solve the inequality.

43. $\frac{5x + 7}{4} \leq -3$

44. $\frac{3x - 2}{5} > -1$

45. $4 \geq \frac{2y - 5}{3} \geq -2$

46. $1 > \frac{3y - 1}{4} > -1$

47. $0 \leq 2z + 5 < 8$

48. $-6 < 5t - 1 < 0$

49. $\frac{x - 5}{4} + \frac{3 - 2x}{3} < -2$

50. $\frac{3 - x}{2} + \frac{5x - 2}{3} < -1$

51. $\frac{2y - 3}{2} + \frac{3y - 1}{5} < y - 1$

52. $\frac{3 - 4y}{6} - \frac{2y - 3}{8} \geq 2 - y$

53. $\frac{1}{2}(x - 4) - 2x \leq 5(3 - x)$

54. $\frac{1}{2}(x + 3) + 2(x - 4) < \frac{1}{3}(x - 3)$

In Exercises 55–58, find the solutions of the equation or inequality displayed in Figure P.20.

55. $x^2 - 2x < 0$ 56. $x^2 - 2x = 0$
 57. $x^2 - 2x > 0$ 58. $x^2 - 2x \leq 0$

X	Y_1	
0	0	
1	-1	
2	0	
3	3	
4	8	
5	15	
6	24	

$Y_1 = X^2 - 2X$

FIGURE P.20 The second column gives values of $y_1 = x^2 - 2x$ for $x = 0, 1, 2, 3, 4, 5,$ and 6 .

59. **Writing to Learn** Explain how the second equation was obtained from the first.

$$x - 3 = 2x + 3, \quad 2x - 6 = 4x + 6$$

60. **Writing to Learn** Explain how the second equation was obtained from the first.

$$2x - 1 = 2x - 4, \quad x - \frac{1}{2} = x - 2$$

61. **Group Activity** Determine whether the two equations are equivalent.

- (a) $3x = 6x + 9, \quad x = 2x + 9$
 (b) $6x + 2 = 4x + 10, \quad 3x + 1 = 2x + 5$

62. **Group Activity** Determine whether the two equations are equivalent.

- (a) $3x + 2 = 5x - 7, \quad -2x + 2 = -7$
 (b) $2x + 5 = x - 7, \quad 2x = x - 7$

Standardized Test Questions

63. **True or False** $-6 > -2$. Justify your answer.
 64. **True or False** $2 \leq \frac{6}{3}$. Justify your answer.

In Exercises 65–68, you may use a graphing calculator to solve these problems.

65. **Multiple Choice** Which of the following equations is equivalent to the equation $3x + 5 = 2x + 1$?
 (A) $3x = 2x$ (B) $3x = 2x + 4$
 (C) $\frac{3}{2}x + \frac{5}{2} = x + 1$ (D) $3x + 6 = 2x$
 (E) $3x = 2x - 4$
66. **Multiple Choice** Which of the following inequalities is equivalent to the inequality $-3x < 6$?
 (A) $3x < -6$ (B) $x < 10$
 (C) $x > -2$ (D) $x > 2$
 (E) $x > 3$

67. **Multiple Choice** Which of the following is the solution to the equation $x(x + 1) = 0$?

- (A) $x = 0$ or $x = -1$ (B) $x = 0$ or $x = 1$
 (C) only $x = -1$ (D) only $x = 0$
 (E) only $x = 1$

68. **Multiple Choice** Which of the following represents an equation equivalent to the equation

$$\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{1}{3}$$

that is cleared of fractions?

- (A) $2x + 1 = x - 1$ (B) $8x + 6 = 3x - 4$
 (C) $4x + 3 = \frac{3}{2}x - 2$ (D) $4x + 3 = 3x - 4$
 (E) $4x + 6 = 3x - 4$

Explorations

69. **Testing Inequalities on a Calculator**

- (a) The calculator we use indicates that the statement $2 < 3$ is true by returning the value 1 (for true) when $2 < 3$ is entered. Try it with your calculator.
 (b) The calculator we use indicates that the statement $2 < 1$ is false by returning the value 0 (for false) when $2 < 1$ is entered. Try it with your calculator.
 (c) Use your calculator to test which of these two numbers is larger: $799/800, 800/801$.
 (d) Use your calculator to test which of these two numbers is larger: $-102/101, -103/102$.
 (e) If your calculator returns 0 when you enter $2x + 1 < 4$, what can you conclude about the value stored in x ?

Extending the Ideas

70. **Perimeter of a Rectangle** The formula for the perimeter P of a rectangle is

$$P = 2(L + W).$$

Solve this equation for W .

71. **Area of a Trapezoid** The formula for the area A of a trapezoid is

$$A = \frac{1}{2}h(b_1 + b_2).$$

Solve this equation for b_1 .

72. **Volume of a Sphere**

The formula for the volume V of a sphere is

$$V = \frac{4}{3}\pi r^3.$$

Solve this equation for r .



73. **Celsius and Fahrenheit** The formula for Celsius temperature in terms of Fahrenheit temperature is

$$C = \frac{5}{9}(F - 32)$$

Solve the equation for F .

P.4

Lines in the Plane

What you'll learn about

- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables

... and why

Linear equations are used extensively in applications involving business and behavioral science.

Slope of a Line

The slope of a nonvertical line is the ratio of the amount of vertical change to the amount of horizontal change between two points. For the points (x_1, y_1) and (x_2, y_2) , the vertical change is $\Delta y = y_2 - y_1$ and the horizontal change is $\Delta x = x_2 - x_1$. Δy is read “delta” y . See Figure P.21.

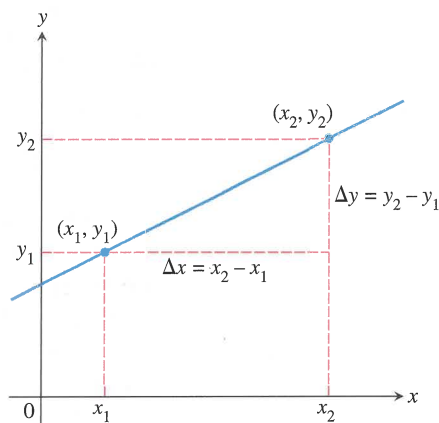


FIGURE P.21 The slope of a nonvertical line can be found from the coordinates of any two points of the line.

DEFINITION Slope of a Line

The **slope** of the nonvertical line through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

If the line is vertical, then $x_1 = x_2$ and the slope is undefined.

SLOPE FORMULA

The slope does not depend on the order of the points. We could use $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 2)$ in Example 1a. Check it out.

EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line through the two points. Sketch a graph of the line.

- (a) $(-1, 2)$ and $(4, -2)$ (b) $(1, 1)$, and $(3, 4)$

SOLUTION

- (a) The two points are $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (4, -2)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - 2}{4 - (-1)} = -\frac{4}{5}.$$

continued

(b) The two points are $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (3, 4)$. Thus,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 1} = \frac{3}{2}.$$

The graphs of these two lines are shown in Figure P.22.

Now try Exercise 3.

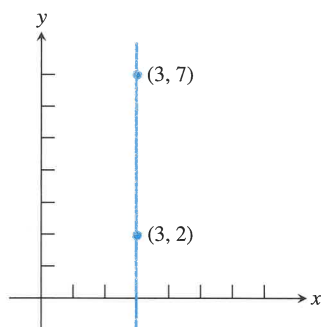


FIGURE P.23 Applying the slope formula to this vertical line gives $m = 5/0$, which is not defined. Thus, the slope of a vertical line is undefined.

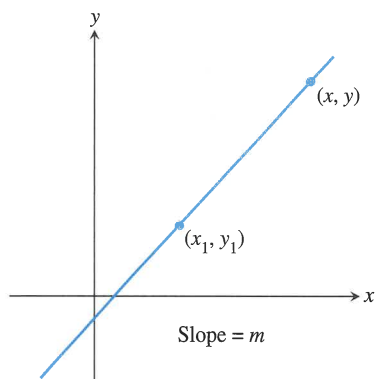


FIGURE P.24 The line through (x_1, y_1) with slope m .

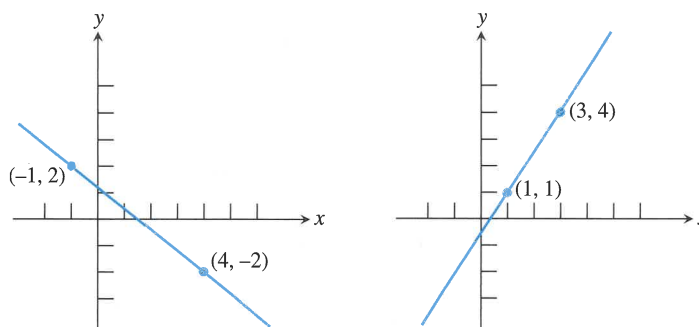


FIGURE P.22 The graphs of the two lines in Example 1.

Figure P.23 shows a vertical line through the points $(3, 2)$ and $(3, 7)$. If we try to calculate its slope using the slope formula $(y_2 - y_1)/(x_2 - x_1)$, we get zero in the denominator. So, it makes sense to say that a vertical line does not have a slope, or that its slope is undefined.

Point-Slope Form Equation of a Line

If we know the coordinates of one point on a line and the slope of the line, then we can find an equation for that line. For example, the line in Figure P.24 passes through the point (x_1, y_1) and has slope m . If (x, y) is any other point on this line, the definition of the slope yields the equation

$$m = \frac{y - y_1}{x - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1).$$

An equation written this way is in the *point-slope form*.

DEFINITION Point-Slope Form of an Equation of a Line

The **point-slope form** of an equation of a line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 Using the Point-Slope Form

Use the point-slope form to find an equation of the line that passes through the point $(-3, -4)$ and has slope 2.

continued

y-INTERCEPT

The b in $y = mx + b$ is often referred to as “the y -intercept” instead of “the y -coordinate of the y -intercept.”

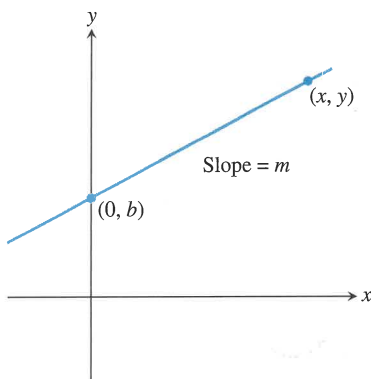


FIGURE P.25 The line with slope m and y -intercept $(0, b)$.

SOLUTION Substitute $x_1 = -3$, $y_1 = -4$, and $m = 2$ into the point-slope form, and simplify the resulting equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope form} \\ y - (-4) &= 2(x - (-3)) && x_1 = -3, y_1 = -4, m = 2 \\ y + 4 &= 2x - 2(-3) && \text{Distributive property} \\ y + 4 &= 2x + 6 \\ y &= 2x + 2 && \text{A common simplified form} \end{aligned}$$

Now try Exercise 11.

Slope-Intercept Form Equation of a Line

The **y -intercept** of a nonvertical line is the point where the line intersects the y -axis. If we know the y -intercept and the slope of the line, we can apply the point-slope form to find an equation of the line.

Figure P.25 shows a line with slope m and y -intercept $(0, b)$. A point-slope form equation for this line is $y - b = m(x - 0)$. By rewriting this equation we obtain the form known as the *slope-intercept form*.

DEFINITION Slope-Intercept Form of an Equation of a Line

The **slope-intercept form** of an equation of a line with slope m and y -intercept $(0, b)$ is

$$y = mx + b.$$

EXAMPLE 3 Using the Slope-Intercept Form

Write an equation of the line with slope 3 that passes through the point $(-1, 6)$ using the slope-intercept form.

SOLUTION

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ y &= 3x + b && m = 3 \\ 6 &= 3(-1) + b && y = 6 \text{ when } x = -1 \\ b &= 9 \end{aligned}$$

The slope-intercept form of the equation is $y = 3x + 9$.

Now try Exercise 21.

We cannot use the phrase “the equation of a line” because each line has many different equations. Every line has an equation that can be written in the form $Ax + By + C = 0$ where A and B are not both zero. This form is the **general form** for an equation of a line.

If $B \neq 0$, the general form can be changed to the slope-intercept form as follows:

$$Ax + By + C = 0$$

$$By = -Ax - C$$

$$y = \underbrace{-\frac{A}{B}}_{\text{slope}}x + \underbrace{\left(-\frac{C}{B}\right)}_{\text{y-intercept}}$$

Forms of Equations of Lines

General form: $Ax + By + C = 0$, A and B not both zero

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: $x = a$

Horizontal line: $y = b$

Graphing Linear Equations in Two Variables

A **linear equation in x and y** is one that can be written in the form

$$Ax + By = C,$$

where A and B are not both zero. Rewriting this equation in the form $Ax + By - C = 0$ we see that it is the general form of an equation of a line. If $B = 0$, the line is vertical, and if $A = 0$, the line is horizontal.

The **graph** of an equation in x and y consists of all pairs (x, y) that are solutions of the equation. For example, $(1, 2)$ is a **solution** of the equation $2x + 3y = 8$ because substituting $x = 1$ and $y = 2$ into the equation leads to the true statement $8 = 8$. The pairs $(-2, 4)$ and $(2, 4/3)$ are also solutions.

Because the graph of a linear equation in x and y is a straight line, we need only to find two solutions and then connect them with a straight line to draw its graph. If a line is neither horizontal nor vertical, then two easy points to find are its x -intercept and the y -intercept. The **x -intercept** is the point $(x', 0)$ where the graph intersects the x -axis. Set $y = 0$ and solve for x to find the x -intercept. The coordinates of the y -intercept are $(0, y')$. Set $x = 0$ and solve for y to find the y -intercept.

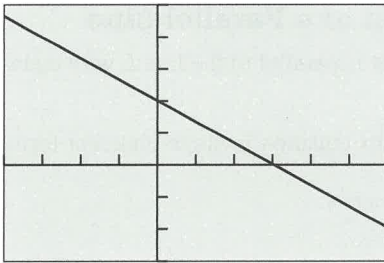
WINDOW
 $X_{\min}=-10$
 $X_{\max}=10$
 $X_{\text{scl}}=1$
 $Y_{\min}=-10$
 $Y_{\max}=10$
 $Y_{\text{scl}}=1$
 $X_{\text{res}}=1$

FIGURE P.26 The window dimensions for the *standard window*. The notation “[$-10, 10$] by [$-10, 10$]” is used to represent window dimensions like these.

Graphing with a Graphing Utility

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form $y =$ (an expression in x).
2. Enter the equation into the grapher.
3. Select an appropriate **viewing window** (see Figure P.26).
4. Press the “graph” key.



$[-4, 6]$ by $[-3, 5]$

FIGURE P.27 The graph of $2x + 3y = 6$. The points $(0, 2)$ (y -intercept) and $(3, 0)$ (x -intercept) appear to lie on the graph and, as pairs, are solutions of the equation, providing visual support that the graph is correct. (Example 4)

VIEWING WINDOW

The viewing window $[-4, 6]$ by $[-3, 5]$ in Figure P.27 means $-4 \leq x \leq 6$ and $-3 \leq y \leq 5$.

SQUARE VIEWING WINDOW

A **square viewing window** on a grapher is one in which angles appear to be true. For example, the line $y = x$ will appear to make a 45° angle with the positive x -axis. Furthermore, a distance of 1 on the x - and y -axes will appear to be the same. That is, if $Xscl = Yscl$, the distance between consecutive tick marks on the x - and y -axes will appear to be the same.

A graphing utility, often referred to as a *grapher*, computes y -values for a select set of x -values between $Xmin$ and $Xmax$ and plots the corresponding (x, y) points.

EXAMPLE 4 Use a Graphing Utility

Draw the graph of $2x + 3y = 6$.

SOLUTION First we solve for y .

$$2x + 3y = 6$$

$$3y = -2x + 6 \quad \text{Solve for } y.$$

$$y = -\frac{2}{3}x + 2 \quad \text{Divide by 3.}$$

Figure P.27 shows the graph of $y = -(2/3)x + 2$, or equivalently, the graph of the linear equation $2x + 3y = 6$ in the $[-4, 6]$ by $[-3, 5]$ viewing window.

Now try Exercise 27.

Parallel and Perpendicular Lines

EXPLORATION 1 Investigating Graphs of Linear Equations

1. What do the graphs of $y = mx + b$ and $y = mx + c$, $b \neq c$, have in common? How are they different?
2. Graph $y = 2x$ and $y = -(1/2)x$ in a *square viewing window* (see margin note). On the calculator we use, the “decimal window” $[-4.7, 4.7]$ by $[-3.1, 3.1]$ is square. Estimate the angle between the two lines.
3. Repeat part 2 for $y = mx$ and $y = -(1/m)x$ with $m = 1, 3, 4$, and 5 .

Parallel lines and perpendicular lines were involved in Exploration 1. Using a grapher to decide when lines are parallel or perpendicular is risky. Here is an algebraic test to determine when two lines are parallel or perpendicular.

Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if their slopes are equal.
2. Two nonvertical lines are perpendicular if and only if their slopes m_1 and m_2 are opposite reciprocals. That is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 5 Finding an Equation of a Parallel Line

Find an equation of the line through $P(1, -2)$ that is parallel to the line L with equation $3x - 2y = 1$.

SOLUTION We find the slope of L by writing its equation in slope-intercept form.

$$\begin{aligned} 3x - 2y &= 1 && \text{Equation for } L \\ -2y &= -3x + 1 && \text{Subtract } 3x. \\ y &= \frac{3}{2}x - \frac{1}{2} && \text{Divide by } -2. \end{aligned}$$

The slope of L is $3/2$.

The line whose equation we seek has slope $3/2$ and contains the point $(x_1, y_1) = (1, -2)$. Thus, the point-slope form equation for the line we seek is

$$\begin{aligned} y + 2 &= \frac{3}{2}(x - 1) \\ y + 2 &= \frac{3}{2}x - \frac{3}{2} && \text{Distributive property} \\ y &= \frac{3}{2}x - \frac{7}{2} \end{aligned}$$

Now try Exercise 41(a).

EXAMPLE 6 Finding an Equation of a Perpendicular Line

Find an equation of the line through $P(2, -3)$ that is perpendicular to the line L with equation $4x + y = 3$. Support the result with a grapher.

SOLUTION We find the slope of L by writing its equation in slope-intercept form.

$$\begin{aligned} 4x + y &= 3 && \text{Equation for } L \\ y &= -4x + 3 && \text{Subtract } 4x. \end{aligned}$$

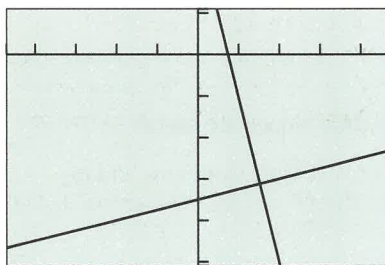
The slope of L is -4 .

The line whose equation we seek has slope $-1/(-4) = 1/4$ and passes through the point $(x_1, y_1) = (2, -3)$. Thus, the point-slope form equation for the line we seek is

$$\begin{aligned} y - (-3) &= \frac{1}{4}(x - 2) \\ y + 3 &= \frac{1}{4}x - \frac{2}{4} && \text{Distributive property} \\ y &= \frac{1}{4}x - \frac{7}{2} \end{aligned}$$

Figure P.28 shows the graphs of the two equations in a square viewing window and suggests that the graphs are perpendicular.

Now try Exercise 43(b).



$[-4.7, 4.7]$ by $[-5.1, 1.1]$

FIGURE P.28 The graphs of $y = -4x + 3$ and $y = (1/4)x - 7/2$ in this square viewing window appear to intersect at a right angle. (Example 6)

Applying Linear Equations in Two Variables

Linear equations and their graphs occur frequently in applications. Algebraic solutions to these application problems often require finding an equation of a line and solving a linear equation in one variable. Grapher techniques complement algebraic ones.

EXAMPLE 7 Finding the Depreciation of Real Estate

Camelot Apartments purchased a \$50,000 building and depreciates it \$2000 per year over a 25-year period.

- (a) Write a linear equation giving the value y of the building in terms of the years x after the purchase.
- (b) In how many years will the value of the building be \$24,500?

SOLUTION

- (a) We need to determine the value of m and b so that $y = mx + b$, where $0 \leq x \leq 25$. We know that $y = 50,000$ when $x = 0$, so the line has y -intercept $(0, 50,000)$ and $b = 50,000$. One year after purchase ($x = 1$), the value of the building is $50,000 - 2,000 = 48,000$. So when $x = 1$, $y = 48,000$. Using algebra, we find

$$y = mx + b$$

$$48,000 = m \cdot 1 + 50,000 \quad y = 48,000 \text{ when } x = 1$$

$$-2000 = m$$

The value y of the building x years after its purchase is

$$y = -2000x + 50,000.$$

- (b) We need to find the value of x when $y = 24,500$.

$$y = -2000x + 50,000$$

Again, using algebra we find

$$24,500 = -2000x + 50,000 \quad \text{Set } y = 24,500.$$

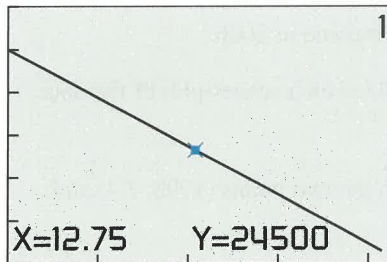
$$-25,500 = -2000x \quad \text{Subtract } 50,000.$$

$$12.75 = x$$

The depreciated value of the building will be \$24,500 exactly 12.75 years, or 12 years 9 months, after purchase by Camelot Apartments. We can support our algebraic work both graphically and numerically. The trace coordinates in Figure P.29a show graphically that $(12.75, 24,500)$ is a solution of $y = -2000x + 50,000$. This means that $y = 24,500$ when $x = 12.75$.

Figure P.29b is a table of values for $y = -2000x + 50,000$ for a few values of x . The fourth line of the table shows numerically that $y = 24,500$ when $x = 12.75$.

Now try Exercise 45.



$[0, 23.5]$ by $[0, 60000]$

(a)

X	Y
12	26000
12.25	25500
12.5	25000
12.75	24500
13	24000
13.25	23500
13.5	23000

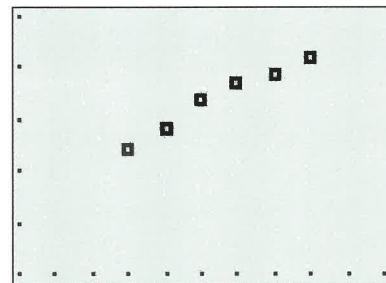
$Y_1 = -2000X + 50000$

(b)

FIGURE P.29 A (a) graph and (b) table of values for $y = -2000x + 50,000$. (Example 7)

Figure P.30 on page 38 shows Americans' income from 1998 to 2003 in trillions of dollars and a corresponding scatter plot of the data. In Example 8 we model the data in Figure P.30 with a linear equation.

Year	Amount (trillions of dollars)
1998	7.4
1999	7.8
2000	8.4
2001	8.7
2002	8.9
2003	9.2



[1995, 2005] by [5, 10]

FIGURE P.30 Americans' Personal Income.

Source: U.S. Census Bureau, *Statistical Abstract of the United States, 2004–2005*. (Example 8)



EXAMPLE 8 Finding a Linear Model for Americans' Personal Income

American's personal income in trillions of dollars is given in Figure P.30.

- Write a linear equation for Americans' income y in terms of the year x using the points (1998, 7.4) and (1999, 7.8).
- Use the equation in (a) to estimate Americans' income in 2001.
- Use the equation in (a) to predict Americans' income in 2006.
- Superimpose a graph of the linear equation in (a) on a scatter plot of the data.

SOLUTION

- Let $y = mx + b$. The slope of the line through the two points (1998, 7.4) and (1999, 7.8) is

$$m = \frac{7.8 - 7.4}{1999 - 1998} = 0.4.$$

The value of 7.4 trillion dollars in 1998 gives $y = 7.4$ when $x = 1998$.

$$y = mx + b$$

$$y = 0.4x + b \quad m = 0.4$$

$$7.4 = 0.4(1998) + b \quad y = 7.4 \text{ when } x = 1998$$

$$b = 7.4 - 0.4(1998)$$

$$b = -791.8$$

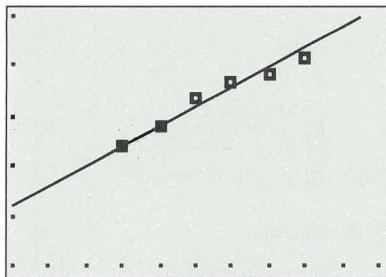
∴ The linear equation we seek is $y = 0.4x - 791.8$.

- We need to find the value of y when $x = 2001$.

$$y = 0.4x - 791.8$$

$$y = 0.4(2001) - 791.8 \quad \text{Set } x = 2001.$$

$$y = 8.6$$



[1995, 2005] by [5, 10]

FIGURE P.31 Linear model for Americans' personal income. (Example 8)

Using the linear model we found in (a) we estimate Americans' income in 2001 to be 8.6 trillion dollars, a little less than the actual amount 8.7 trillion.

(c) We need to find the value of y when $x = 2006$.

$$y = 0.4x - 791.8$$

$$y = 0.4(2006) - 791.8 \quad \text{Set } x = 2006.$$

$$y = 10.6$$

Using the linear model we found in (a) we predict Americans' income in 2006 to be 10.6 trillion dollars.

(d) The graph and scatter plot are shown in Figure P.31.

Now try Exercise 51.

CHAPTER OPENER PROBLEM (from page 1)

PROBLEM: Assume that the speed of light is approximately 186,000 miles per second. (It took a long time to arrive at this number. See the note below about the speed of light.)

- If the distance from the Moon to the Earth is approximately 237,000 miles, find the length of time required for light to travel from the Earth to the Moon.
- If light travels from the Earth to the Sun in 8.32 minutes, approximate the distance from the Earth to the Sun.
- If it takes 5 hours and 29 seconds for light to travel from the Sun to Pluto, approximate the distance from the Sun to Pluto.

SOLUTION: We use the linear equation $d = r \times t$ (distance = rate \times time) to make the calculations with $r = 186,000$ miles/second.

(a) Here $d = 237,000$ miles, so

$$t = \frac{d}{r} = \frac{237,000 \text{ miles}}{186,000 \text{ miles/second}} \approx 1.27 \text{ seconds.}$$

The length of time required for light to travel from the Earth to the Moon is about 1.27 seconds.

(b) Here $t = 8.32$ minutes = 499.2 seconds, so

$$d = r \times t = 186,000 \frac{\text{miles}}{\text{second}} \times 499.2 \text{ seconds} = 92,851,200 \text{ miles.}$$

The distance from the Earth to the Sun is about 93 million miles.

(c) Here $t = 5$ hours and 29 minutes = 329 minutes = 19,740 seconds, so

$$\begin{aligned} d &= r \times t = 186,000 \frac{\text{miles}}{\text{second}} \times 19,740 \text{ seconds} \\ &= 3,671,640,000 \text{ miles.} \end{aligned}$$

The distance from the Sun to Pluto is about 3.7×10^9 miles.

THE SPEED OF LIGHT

Many scientists have tried to measure the speed of light. For example, Galileo Galilei (1564–1642) attempted to measure the speed of light without much success. Visit the following web site for some interesting information about this topic: <http://www.what-is-the-speed-of-light.com/>

QUICK REVIEW P.4

In Exercises 1–4, solve for x .

1. $-75x + 25 = 200$
2. $400 - 50x = 150$
3. $3(1 - 2x) + 4(2x - 5) = 7$
4. $2(7x + 1) = 5(1 - 3x)$

In Exercises 5–8, solve for y .

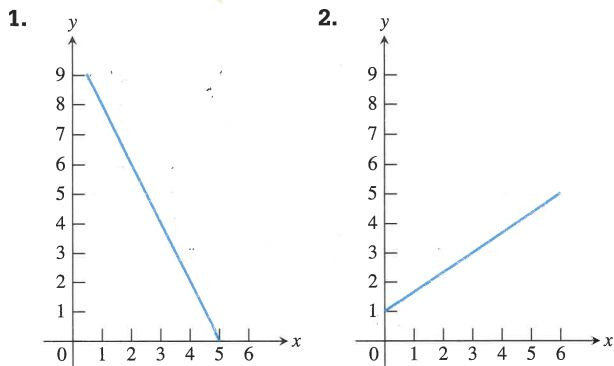
5. $2x - 5y = 21$
6. $\frac{1}{3}x + \frac{1}{4}y = 2$
7. $2x + y = 17 + 2(x - 2y)$
8. $x^2 + y = 3x - 2y$

In Exercises 9 and 10, simplify the fraction.

9. $\frac{9 - 5}{-2 - (-8)}$
10. $\frac{-4 - 6}{-14 - (-2)}$

SECTION P.4 EXERCISES

In Exercises 1 and 2, estimate the slope of the line.



In Exercises 3–6, find the slope of the line through the pair of points.

3. $(-3, 5)$ and $(4, 9)$
4. $(-2, 1)$ and $(5, -3)$
5. $(-2, -5)$ and $(-1, 3)$
6. $(5, -3)$ and $(-4, 12)$

In Exercises 7–10, find the value of x or y so that the line through the pair of points has the given slope.

Points	Slope
7. $(x, 3)$ and $(5, 9)$	$m = 2$
8. $(-2, 3)$ and $(4, y)$	$m = -3$
9. $(-3, -5)$ and $(4, y)$	$m = 3$
10. $(-8, -2)$ and $(x, 2)$	$m = 1/2$

In Exercises 11–14, find a *point-slope form* equation for the line through the point with given slope.

Point	Slope	Point	Slope
11. $(1, 4)$	$m = 2$	12. $(-4, 3)$	$m = -2/3$
13. $(5, -4)$	$m = -2$	14. $(-3, 4)$	$m = 3$

In Exercises 15–20, find a *general form equation* for the line through the pair of points.

15. $(-7, -2)$ and $(1, 6)$
16. $(-3, -8)$ and $(4, -1)$
17. $(1, -3)$ and $(5, -3)$
18. $(-1, -5)$ and $(-4, -2)$
19. $(-1, 2)$ and $(2, 5)$
20. $(4, -1)$ and $(4, 5)$

In Exercises 21–26, find a slope-intercept form equation for the line.

21. The line through $(0, 5)$ with slope $m = -3$
22. The line through $(1, 2)$ with slope $m = 1/2$
23. The line through the points $(-4, 5)$ and $(4, 3)$
24. The line through the points $(4, 2)$ and $(-3, 1)$
25. The line $2x + 5y = 12$

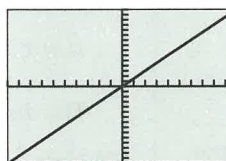
26. The line $7x - 12y = 96$

In Exercises 27–30, graph the linear equation on a grapher. Choose a viewing window that shows the line intersecting both the x - and y -axes.

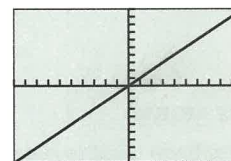
27. $8x + y = 49$
28. $2x + y = 35$
29. $123x + 7y = 429$
30. $2100x + 12y = 3540$

In Exercises 31 and 32, the line contains the origin and the point in the upper right corner of the grapher screen.

31. **Writing to Learn** Which line shown here has the greater slope? Explain.

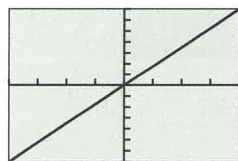


$[-10, 10]$ by $[-15, 15]$
(a)



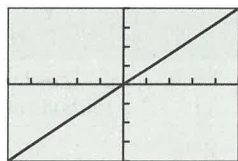
$[-10, 10]$ by $[-10, 10]$
(b)

- 32. Writing to Learn** Which line shown here has the greater slope? Explain.



$[-20, 20]$ by $[-35, 35]$

(a)



$[-5, 5]$ by $[-20, 20]$

(b)

In Exercises 33–36, find the value of x and the value of y for which $(x, 14)$ and $(18, y)$ are points on the graph.

33. $y = 0.5x + 12$

34. $y = -2x + 18$

35. $3x + 4y = 26$

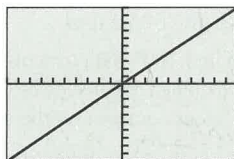
36. $3x - 2y = 14$

In Exercises 37–40, find the values for Y_{min} , Y_{max} , and Y_{scl} that will make the graph of the line appear in the viewing window as shown here.

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=
Ymax=
Yscl=
Xres=1

```



37. $y = 3x$

38. $y = 5x$

39. $y = \frac{2}{3}x$

40. $y = \frac{5}{4}x$

In Exercises 41–44, (a) find an equation for the line passing through the point and parallel to the given line, and (b) find an equation for the line passing through the point and perpendicular to the given line. Support your work graphically.

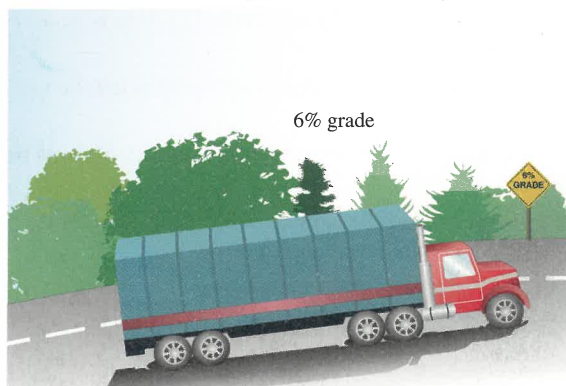
Point	Line
41. (1, 2)	$y = 3x - 2$
42. (-2, 3)	$y = -2x + 4$
43. (3, 1)	$2x + 3y = 12$
44. (6, 1)	$3x - 5y = 15$

- 45. Real Estate Appreciation** Bob Michaels purchased a house 8 years ago for \$42,000. This year it was appraised at \$67,500.

- (a) A linear equation $V = mt + b$, $0 \leq t \leq 15$, represents the value V of the house for 15 years after it was purchased. Determine m and b .
- (b) Graph the equation and trace to estimate in how many years after purchase this house will be worth \$72,500.
- (c) Write and solve an equation algebraically to determine how many years after purchase this house will be worth \$74,000.
- (d) Determine how many years after purchase this house will be worth \$80,250.

- 46. Investment Planning** Mary Ellen plans to invest \$18,000, putting part of the money x into a savings that pays 5% annually and the rest into an account that pays 8% annually.
- (a) What are the possible values of x in this situation?
- (b) If Mary Ellen invests x dollars at 5%, write an equation that describes the total interest I received from both accounts at the end of one year.
- (c) Graph and trace to estimate how much Mary Ellen invested at 5% if she earned \$1020 in total interest at the end of the first year.
- (d) Use your grapher to generate a table of values for I to find out how much Mary Ellen should invest at 5% to earn \$1185 in total interest in one year.
- 47. Navigation** A commercial jet airplane climbs at takeoff with slope $m = 3/8$. How far in the horizontal direction will the airplane fly to reach an altitude of 12,000 ft above the takeoff point?

- 48. Grade of a Highway** Interstate 70 west of Denver, Colorado, has a section posted as a 6% grade. This means that for a horizontal change of 100 ft there is a 6-ft vertical change.



- (a) Find the slope of this section of the highway.
- (b) On a highway with a 6% grade what is the horizontal distance required to climb 250 ft?
- (c) A sign along the highway says 6% grade for the next 7 mi. Estimate how many feet of vertical change there are along those next 7 mi. (There are 5280 ft in 1 mile.)
- 49. Writing to Learn Building Specifications** Asphalt shingles do not meet code specifications on a roof that has less than a 4-12 pitch. A 4-12 pitch means there are 4 ft of vertical change in 12 ft of horizontal change. A certain roof has slope $m = 3/8$. Could asphalt shingles be used on that roof? Explain.
- 50. Revisiting Example 8** Use the linear equation found in Example 8 to estimate Americans' income in 2000, 2002, 2003 displayed in Figure P.30.

51. Americans' Spending Americans personal consumption expenditures from 1998 to 2003 in trillions of dollars is shown in the table.

x	1998	1999	2000	2001	2002	2003
y	5.9	6.3	6.7	7.0	7.4	7.8

- (a) Write a linear equation for Americans' spending y in terms of the year x using the points (1998, 5.9) and (1999, 6.3).
- (b) Use the equation in (a) to estimate Americans' expenditures in 2002.
- (c) Use the equation in (a) to predict Americans' expenditures in 2006.
- (d) Superimpose a graph of the linear equation in (a) on a scatter plot of the data.

52. U.S. Imports from Mexico The total y in billions of dollars of U.S. imports from Mexico for each year x from 1996 to 2003 is given in the table. (Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.)

x	1996	1997	1998	1999	2000	2001	2002	2003
y	74.3	85.9	94.6	109.7	135.9	131.3	134.6	138.1

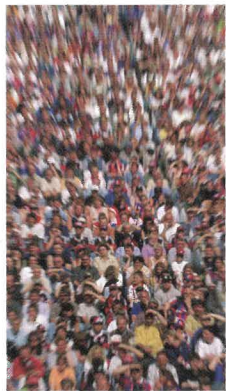
- (a) Use the pairs (1997, 85.9) and (2001, 131.3) to write a linear equation for x and y .
- (b) Superimpose the graph of the linear equation in (a) on a scatter plot of the data.
- (c) Use the equation in (a) to predict the total U.S. imports from Mexico in 2006.

53. World Population The midyear world population for the years 1997 to 2004 (in millions) is shown in Table P.7.



Table P.7 World Population

Year	Population (millions)
1997	5852
1998	5930
1999	6006
2000	6082
2001	6156
2002	6230
2003	6303
2004	6377



Source: <http://www.census.gov/ipc/www/worldpop.html>

- (a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Draw a scatter plot of the data.
- (b) Use the 1997 and 2004 data to write a linear equation for the population y in terms of the year x . Superimpose the graph of the linear equation on the scatter plot of the data.
- (c) Use the equation in (b) to predict the midyear world population in 2006. Compare it with the Census Bureau estimate of 6525.

54. U.S. Exports to Japan The total in billions of dollars of U.S. exports to Japan from 1996 to 2003 is given in Table P.8.



Table P.8 U.S. Exports to Japan

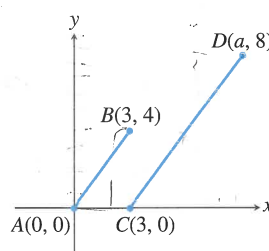
Year	U.S. Exports (billions of dollars)
1996	67.6
1997	65.5
1998	57.8
1999	57.5
2000	64.9
2001	57.4
2002	51.4
2003	52.1

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.

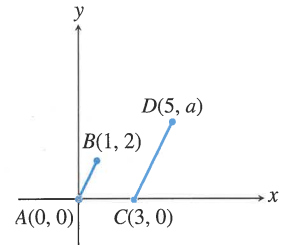
- (a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Draw a scatter plot of the data.
- (b) Use the 1996 and 2003 data to write a linear equation for the U.S. exports to Japan y in terms of the year x . Superimpose the graph of the linear equation on the scatter plot in (a).
- (c) Use the equation in (b) to predict the U.S. exports to Japan in 2006.

In Exercises 55 and 56, determine a so that the line segments AB and CD are parallel.

55.

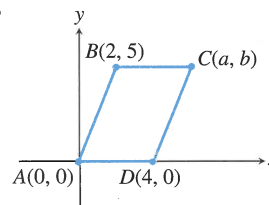


56.

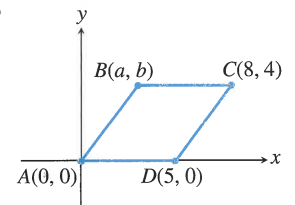


In Exercises 57 and 58, determine a and b so that figure $ABCD$ is a parallelogram.

57.



58.



59. Writing to Learn Perpendicular Lines

- (a) Is it possible for two lines with positive slopes to be perpendicular? Explain.
- (b) Is it possible for two lines with negative slopes to be perpendicular? Explain.

60. Group Activity Parallel and Perpendicular Lines

- (a) Assume that $c \neq d$ and a and b are not both zero. Show that $ax + by = c$ and $ax + by = d$ are parallel lines. Explain why the restrictions on $a, b, c,$ and d are necessary.
- (b) Assume that a and b are not both zero. Show that $ax + by = c$ and $bx - ay = d$ are perpendicular lines. Explain why the restrictions on a and b are necessary.

Standardized Test Questions

- 61. True or False** The slope of a vertical line is zero. Justify your answer.
- 62. True or False** The graph of any equation of the form $ax + by = c$, where a and b are not both zero, is always a line. Justify your answer.

In Exercises 63–66, you may use a graphing calculator to solve these problems.

- 63. Multiple Choice** Which of the following is an equation of the line through the point $(-2, 3)$ with slope 4?
- (A) $y - 3 = 4(x + 2)$ (B) $y + 3 = 4(x - 2)$
 (C) $x - 3 = 4(y + 2)$ (D) $x + 3 = 4(y - 2)$
 (E) $y + 2 = 4(x - 3)$
- 64. Multiple Choice** Which of the following is an equation of the line with slope 3 and y -intercept -2 ?
- (A) $y = 3x + 2$ (B) $y = 3x - 2$
 (C) $y = -2x + 3$ (D) $x = 3y - 2$
 (E) $x = 3y + 2$
- 65. Multiple Choice** Which of the following lines is perpendicular to the line $y = -2x + 5$?
- (A) $y = 2x + 1$ (B) $y = -2x - \frac{1}{5}$
 (C) $y = -\frac{1}{2}x + \frac{1}{3}$ (D) $y = -\frac{1}{2}x + 3$
 (E) $y = \frac{1}{2}x - 3$
- 66. Multiple Choice** Which of the following is the slope of the line through the two points $(-2, 1)$ and $(1, -4)$?
- (A) $-\frac{3}{5}$ (B) $\frac{3}{5}$
 (C) $-\frac{5}{3}$ (D) $\frac{5}{3}$
 (E) -3

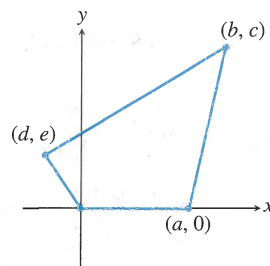
- (c) Draw the graph for $a = 5, b = 3$.
- (d) Use your graphs in (a), (b), (c) to conjecture what a and b represent when $c = 1$. Prove your conjecture.
- (e) Repeat (a)–(d) for $c = 2$.
- (f) If $c = -1$, what do a and b represent?

68. Investigating Graphs of Linear Equations

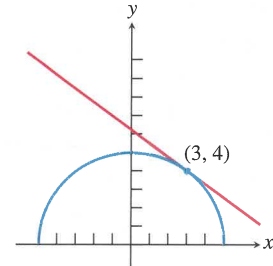
- (a) Graph $y = mx$ for $m = -3, -2, -1, 1, 2, 3$ in the window $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?
- (b) If $m > 0$, what do the graphs of $y = mx$ and $y = -mx$ have in common? How are they different?
- (c) Graph $y = 0.3x + b$ for $b = -3, -2, -1, 0, 1, 2, 3$ in $[-8, 8]$ by $[-5, 5]$. What do these graphs have in common? How are they different?

Extending the Ideas

- 69. Connecting Algebra and Geometry** Show that if the midpoints of consecutive sides of any quadrilateral (see figure) are connected, the result is a parallelogram.

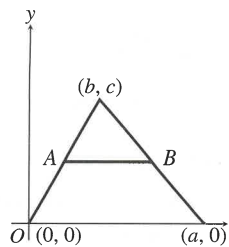


Art for Exercise 69



Art for Exercise 70

- 70. Connecting Algebra and Geometry** Consider the semicircle of radius 5 centered at $(0, 0)$ as shown in the figure. Find an equation of the line tangent to the semicircle at the point $(3, 4)$. (Hint: A line tangent to a circle is perpendicular to the radius at the point of tangency.)
- 71. Connecting Algebra and Geometry** Show that in any triangle (see figure), the line segment joining the midpoints of two sides is parallel to the third side and half as long.

**Explorations**

- 67. Exploring the Graph of $\frac{x}{a} + \frac{y}{b} = c, a \neq 0, b \neq 0$**

Let $c = 1$.

- (a) Draw the graph for $a = 3, b = -2$.
- (b) Draw the graph for $a = -2, b = -3$.

P.5

Solving Equations Graphically, Numerically, and Algebraically

What you'll learn about

- Solving Equations Graphically
- Solving Quadratic Equations
- Approximating Solutions of Equations Graphically
- Approximating Solutions of Equations Numerically with Tables
- Solving Equations by Finding Intersections

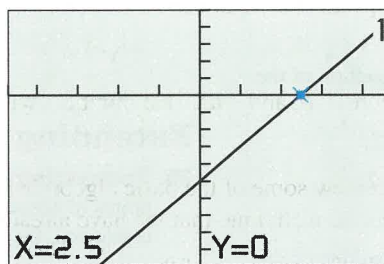
... and why

These basic techniques are involved in using a graphing utility to solve equations in this textbook.

Solving Equations Graphically

The graph of the equation $y = 2x - 5$ (in x and y) can be used to solve the equation $2x - 5 = 0$ (in x). Using the techniques of Section P.3, we can show algebraically that $x = 5/2$ is a solution of $2x - 5 = 0$. Therefore, the ordered pair $(5/2, 0)$ is a solution of $y = 2x - 5$. Figure P.32 suggests that the x -intercept of the graph of the line $y = 2x - 5$ is the point $(5/2, 0)$ as it should be.

One way to solve an equation graphically is to find all its x -intercepts. There are many graphical techniques that can be used to find x -intercepts.



$[-4.7, 4.7]$ by $[-10, 5]$

FIGURE P.32 Using the Trace feature of a grapher, we see that $(2.5, 0)$ is an x -intercept of the graph of $y = 2x - 5$ and, therefore, $x = 2.5$ is a solution of the equation $2x - 5 = 0$.

EXAMPLE 1 Solving by Finding x -Intercepts

Solve the equation $2x^2 - 3x - 2 = 0$ graphically.

SOLUTION

Solve Graphically Find the x -intercepts of the graph of $y = 2x^2 - 3x - 2$ (Figure P.33). We use Trace to see that $(-0.5, 0)$ and $(2, 0)$ are x -intercepts of this graph. Thus, the solutions of this equation are $x = -0.5$ and $x = 2$. Answers obtained graphically are really approximations, although in general they are very good approximations.

Solve Algebraically In this case, we can use factoring to find exact values.

$$2x^2 - 3x - 2 = 0$$

$$(2x + 1)(x - 2) = 0 \quad \text{Factor.}$$

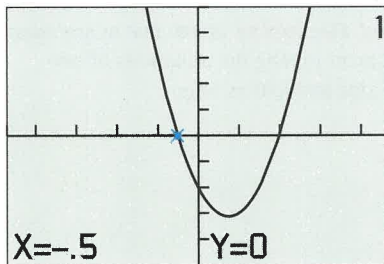
We can conclude that

$$2x + 1 = 0 \quad \text{or} \quad x - 2 = 0,$$

$$x = -1/2 \quad \text{or} \quad x = 2.$$

So, $x = -1/2$ and $x = 2$ are the exact solutions of the original equation.

Now try Exercise 1.



$[-4.7, 4.7]$ by $[-5, 5]$

FIGURE P.33 It appears that $(-0.5, 0)$ and $(2, 0)$ are x -intercepts of the graph of $y = 2x^2 - 3x - 2$. (Example 1)

The algebraic solution procedure used in Example 1 is a special case of the following important property.

Zero Factor Property

Let a and b be real numbers.

$$\text{If } ab = 0, \text{ then } a = 0 \text{ or } b = 0.$$

Solving Quadratic Equations

Linear equations ($ax + b = 0$) and *quadratic equations* are two members of the family of *polynomial equations*, which will be studied in more detail in Chapter 2.

DEFINITION Quadratic Equation in x

A **quadratic equation in x** is one that can be written in the form

$$ax^2 + bx + c = 0,$$

where a , b , and c are real numbers with $a \neq 0$.

We review some of the basic algebraic techniques for solving quadratic equations. One algebraic technique that we have already used in Example 1 is *factoring*.

Quadratic equations of the form $(ax + b)^2 = c$ are fairly easy to solve as illustrated in Example 2.

SQUARE ROOT PRINCIPLE

If $t^2 = K > 0$, then $t = \sqrt{K}$ or $t = -\sqrt{K}$.

EXAMPLE 2 Solving by Extracting Square Roots

Solve $(2x - 1)^2 = 9$ algebraically.

SOLUTION

$$(2x - 1)^2 = 9$$

$$2x - 1 = \pm 3 \quad \text{Extract square roots.}$$

$$2x = 4 \quad \text{or} \quad 2x = -2$$

$$x = 2 \quad \text{or} \quad x = -1$$

Now try Exercise 9.

The technique of Example 2 is more general than you might think because every quadratic equation can be written in the form $(x + b)^2 = c$. The procedure we need to accomplish this is *completing the square*.

Completing the Square

To solve $x^2 + bx = c$ by **completing the square**, add $(b/2)^2$ to both sides of the equation and factor the left side of the new equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}$$

To solve a quadratic equation by completing the square, we simply divide both sides of the equation by the coefficient of x^2 and then complete the square as illustrated in Example 3.

EXAMPLE 3 Solving by Completing the Square

Solve $4x^2 - 20x + 17 = 0$ by completing the square.

SOLUTION

$$4x^2 - 20x + 17 = 0$$

$$x^2 - 5x + \frac{17}{4} = 0 \quad \text{Divide by 4.}$$

$$x^2 - 5x = -\frac{17}{4} \quad \text{Subtract } \left(\frac{17}{4}\right).$$

Completing the square on the equation above we obtain

$$x^2 - 5x + \left(-\frac{5}{2}\right)^2 = -\frac{17}{4} + \left(-\frac{5}{2}\right)^2 \quad \text{Add } \left(-\frac{5}{2}\right)^2.$$

$$\left(x - \frac{5}{2}\right)^2 = 2 \quad \text{Factor and simplify.}$$

$$x - \frac{5}{2} = \pm\sqrt{2} \quad \text{Extract square roots.}$$

$$x = \frac{5}{2} \pm \sqrt{2}$$

$$x = \frac{5}{2} + \sqrt{2} \approx 3.91 \text{ or } x = \frac{5}{2} - \sqrt{2} \approx 1.09$$

Now try Exercise 13.

The procedure of Example 3 can be applied to the general quadratic equation $ax^2 + bx + c = 0$ to produce the following formula for its solutions (see Exercise 68).

Quadratic Formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

EXAMPLE 4 Solving Using the Quadratic Formula

Solve the equation $3x^2 - 6x = 5$.

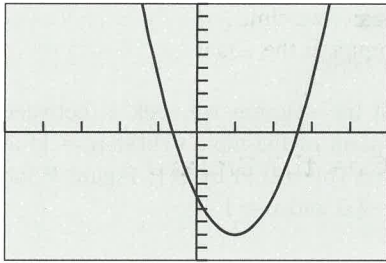
SOLUTION First we subtract 5 from both sides of the equation to put it in the form $ax^2 + bx + c = 0$: $3x^2 - 6x - 5 = 0$. We can see that $a = 3$, $b = -6$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic formula}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-5)}}{2(3)} \quad a = 3, b = -6, c = -5$$

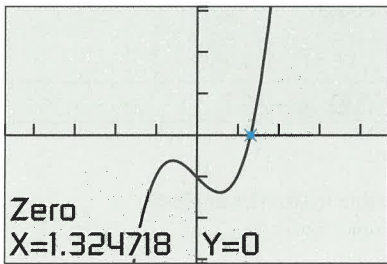
$$x = \frac{6 \pm \sqrt{96}}{6} \quad \text{Simplify.}$$

continued



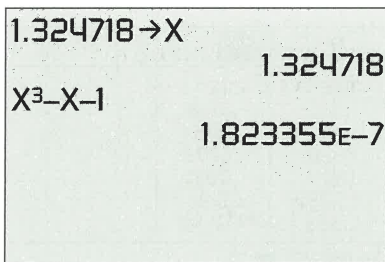
$[-5, 5]$ by $[-10, 10]$

FIGURE P.34 The graph of $y = 3x^2 - 6x - 5$. (Example 4)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a)



(b)

FIGURE P.35 The graph of $y = x^3 - x - 1$. (a) shows that $(1.324718, 0)$ is an approximation to the x -intercept of the graph. (b) supports this conclusion. (Example 5)

$$x = \frac{6 + \sqrt{96}}{6} \approx 2.63 \quad \text{or} \quad x = \frac{6 - \sqrt{96}}{6} \approx -0.63$$

The graph of $y = 3x^2 - 6x - 5$ in Figure P.34 supports that the x -intercepts are approximately -0.63 and 2.63 . **Now try Exercise 19.**

Solving Quadratic Equations Algebraically

There are four basic ways to solve quadratic equations algebraically.

1. **Factoring** (see Example 1)
2. **Extracting Square Roots** (see Example 2)
3. **Completing the Square** (see Example 3)
4. **Using the Quadratic Formula** (see Example 4)

Approximating Solutions of Equations Graphically

A solution of the equation $x^3 - x - 1 = 0$ is a value of x that makes the value of $y = x^3 - x - 1$ equal to zero. Example 5 illustrates a built-in procedure on graphing calculators to find such values of x .

EXAMPLE 5 Solving Graphically

Solve the equation $x^3 - x - 1 = 0$ graphically.

SOLUTION Figure P.35a suggests that $x = 1.324718$ is the solution we seek. Figure P.35b provides numerical support that $x = 1.324718$ is a close approximation to the solution because, when $x = 1.324718$, $x^3 - x - 1 \approx 1.82 \times 10^{-7}$, which is nearly zero. **Now try Exercise 31.**

When solving equations graphically we usually get approximate solutions and not exact solutions. We will use the following agreement about accuracy in this book.

Agreement about Approximate Solutions

For applications, round to a value that is reasonable for the context of the problem.
For all others round to two decimal places unless directed otherwise.

With this accuracy agreement, we would report the solution found in Example 5 as 1.32.

Approximating Solutions of Equations Numerically with Tables

The table feature on graphing calculators provides a numerical *zoom-in procedure* that we can use to find accurate solutions of equations. We illustrate this procedure in Example 6 using the same equation of Example 5.

EXAMPLE 6 Solving Using Tables

Solve the equation $x^3 - x - 1 = 0$ using grapher tables.

SOLUTION From Figure P.35a, we know that the solution we seek is between $x = 1$ and $x = 2$. Figure P.36a sets the starting point of the table (TblStart = 1) at $x = 1$ and increments the numbers in the table ($\Delta\text{Tbl} = 0.1$) by 0.1. Figure P.36b shows that the zero of $x^3 - x - 1$ is between $x = 1.3$ and $x = 1.4$.

TABLE SETUP		
TblStart=1		
$\Delta\text{Tbl}=0.1$		
Indpnt:	Auto	Ask
Depend:	Auto	Ask

(a)

X	Y1	
1	-1	
1.1	-.769	
1.2	-.472	
1.3	-.103	
1.4	.344	
1.5	.875	
1.6	1.496	

Y1 = $X^3 - X - 1$

(b)

FIGURE P.36 (a) gives the setup that produces the table in (b). (Example 6)

The next two steps in this process are shown in Figure P.37.

X	Y1	
1.3	-.103	
1.31	-.0619	
1.32	-.02	
1.33	.02264	
1.34	.0661	
1.35	.11038	
1.36	.15546	

Y1 = $X^3 - X - 1$

(a)

X	Y1	
1.32	-.02	
1.321	-.0158	
1.322	-.0116	
1.323	-.0073	
1.324	-.0031	
1.325	.0012	
1.326	.00547	

Y1 = $X^3 - X - 1$

(b)

FIGURE P.37 In (a) TblStart = 1.3 and $\Delta\text{Tbl} = 0.01$, and in (b) TblStart = 1.32 and $\Delta\text{Tbl} = 0.001$. (Example 6)

From Figure P.37a, we can read that the zero is between $x = 1.32$ and $x = 1.33$; from Figure P.37b, we can read that the zero is between $x = 1.324$ and $x = 1.325$. Because all such numbers round to 1.32, we can report the zero as 1.32 with our accuracy agreement.

Now try Exercise 37.

EXPLORATION 1 Finding Real Zeros of Equations

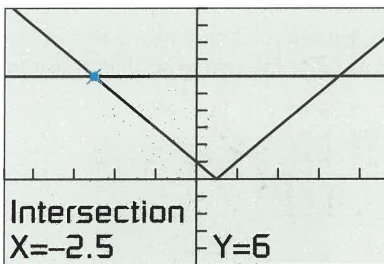
Consider the equation $4x^2 - 12x + 7 = 0$.

1. Use a graph to show that this equation has two real solutions, one between 0 and 1 and the other between 2 and 3.
2. Use the numerical zoom-in procedure illustrated in Example 6 to find each zero accurate to two decimal places.
3. Use the built-in zero finder (see Example 5) to find the two solutions. Then round them to two decimal places.
4. If you are familiar with the graphical zoom-in process, use it to find each solution accurate to two decimal places.
5. Compare the numbers obtained in parts 2, 3, and 4.
6. Support the results obtained in parts 2, 3, and 4 numerically.
7. Use the numerical zoom-in procedure illustrated in Example 6 to find each zero accurate to six decimal places. Compare with the answer found in part 3 with the zero finder.

Solving Equations by Finding Intersections

Sometimes we can rewrite an equation and solve it graphically by finding the *points of intersection* of two graphs. A point (a, b) is a **point of intersection** of two graphs if it lies on both graphs.

We illustrate this procedure with the absolute value equation in Example 7.



$[-4.7, 4.7]$ by $[-5, 10]$

FIGURE P.38 The graphs of $y = |2x - 1|$ and $y = 6$ intersect at $(-2.5, 6)$ and $(3.5, 6)$. (Example 7)

EXAMPLE 7 Solving by Finding Intersections

Solve the equation $|2x - 1| = 6$.

SOLUTION Figure P.38 suggests that the V-shaped graph of $y = |2x - 1|$ intersects the graph of the horizontal line $y = 6$ twice. We can use Trace or the intersection feature of our grapher to see that the two points of intersection have coordinates $(-2.5, 6)$ and $(3.5, 6)$. This means that the original equation has two solutions: -2.5 and 3.5 .

We can use algebra to find the exact solutions. The only two real numbers with absolute value 6 are 6 itself and -6 . So, if $|2x - 1| = 6$, then

$$\begin{aligned} 2x - 1 &= 6 & \text{or} & & 2x - 1 &= -6 \\ x &= \frac{7}{2} = 3.5 & \text{or} & & x &= -\frac{5}{2} = -2.5 \end{aligned}$$

Now try Exercise 39.

QUICK REVIEW P.5

In Exercises 1–4, expand the product.

1. $(3x - 4)^2$
2. $(2x + 3)^2$
3. $(2x + 1)(3x - 5)$
4. $(3y - 1)(5y + 4)$

In Exercises 5–8, factor completely.

5. $25x^2 - 20x + 4$
6. $15x^3 - 22x^2 + 8x$
7. $3x^3 + x^2 - 15x - 5$
8. $y^4 - 13y^2 + 36$

In Exercises 9 and 10, combine the fractions and reduce the resulting fraction to lowest terms.

9. $\frac{x}{2x + 1} - \frac{2}{x + 3}$
10. $\frac{x + 1}{x^2 - 5x + 6} - \frac{3x + 11}{x^2 - x - 6}$

SECTION P.5 EXERCISES

In Exercises 1–6, solve the equation graphically by finding x -intercepts. Confirm by using factoring to solve the equation.

1. $x^2 - x - 20 = 0$
2. $2x^2 + 5x - 3 = 0$
3. $4x^2 - 8x + 3 = 0$
4. $x^2 - 8x = -15$
5. $x(3x - 7) = 6$
6. $x(3x + 11) = 20$

In Exercises 7–12, solve the equation by extracting square roots.

7. $4x^2 = 25$
8. $2(x - 5)^2 = 17$
9. $3(x + 4)^2 = 8$
10. $4(u + 1)^2 = 18$
11. $2y^2 - 8 = 6 - 2y^2$
12. $(2x + 3)^2 = 169$

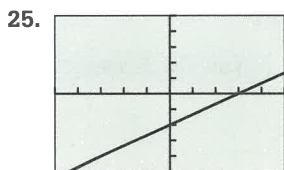
In Exercises 13–18, solve the equation by completing the square.

13. $x^2 + 6x = 7$
14. $x^2 + 5x - 9 = 0$
15. $x^2 - 7x + \frac{5}{4} = 0$
16. $4 - 6x = x^2$
17. $2x^2 - 7x + 9 = (x - 3)(x + 1) + 3x$
18. $3x^2 - 6x - 7 = x^2 + 3x - x(x + 1) + 3$

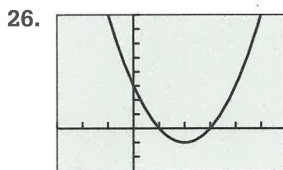
In Exercises 19–24, solve the equation using the quadratic formula.

19. $x^2 + 8x - 2 = 0$
20. $2x^2 - 3x + 1 = 0$
21. $3x + 4 = x^2$
22. $x^2 - 5 = \sqrt{3}x$
23. $x(x + 5) = 12$
24. $x^2 - 2x + 6 = 2x^2 - 6x - 26$

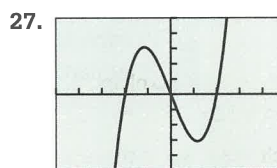
In Exercises 25–28, estimate any x - and y -intercepts that are shown in the graph.



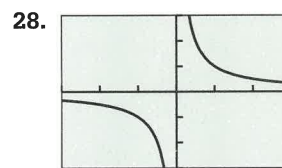
$[-5, 5]$ by $[-5, 5]$



$[-3, 6]$ by $[-3, 8]$



$[-5, 5]$ by $[-5, 5]$



$[-3, 3]$ by $[-3, 3]$

In Exercises 29–34, solve the equation graphically by finding x -intercepts.

29. $x^2 + x - 1 = 0$
30. $4x^2 + 20x + 23 = 0$
31. $x^3 + x^2 + 2x - 3 = 0$
32. $x^3 - 4x + 2 = 0$
33. $x^2 + 4 = 4x$
34. $x^2 + 2x = -2$

In Exercises 35 and 36, the table permits you to estimate a zero of an expression. State the expression and give the zero as accurately as can be read from the table.

35.

X	Y1
4	-.04
41	-.019
42	.0164
43	.0449
44	.0736
45	.1025
46	.1316

Y1 $X^2 + 2X - 1$

36.

X	Y1
-1.735	-.0177
-1.734	-.017
-1.733	-.0057
-1.732	3E-4
-1.731	.0053
-1.73	.0128
-1.729	.0126

Y1 $X^3 - 3X$

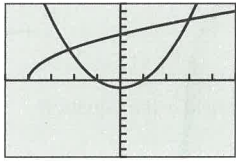
In Exercises 37 and 38, use tables to find the indicated number of solutions of the equation accurate to two decimal places.

37. Two solutions of $x^2 - x - 1 = 0$
38. One solution of $-x^3 + x + 1 = 0$

In Exercises 39–44, solve the equation graphically by finding intersections. Confirm your answer algebraically.

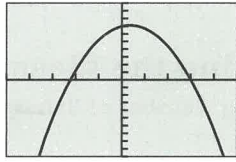
39. $|t - 8| = 2$
40. $|x + 1| = 4$
41. $|2x + 5| = 7$
42. $|3 - 5x| = 4$
43. $|2x - 3| = x^2$
44. $|x + 1| = 2x - 3$

- 45. Interpreting Graphs** The graphs in the two viewing windows shown here can be used to solve the equation $3\sqrt{x+4} = x^2 - 1$ graphically.



$[-5, 5]$ by $[-10, 10]$

(a)



$[-5, 5]$ by $[-10, 10]$

(b)

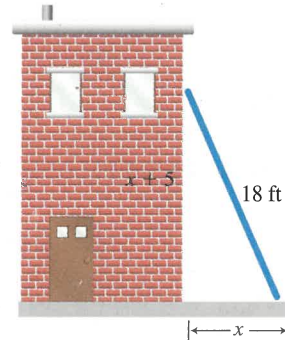
- (a) The viewing window in (a) illustrates the intersection method for solving. Identify the two equations that are graphed.
- (b) The viewing window in (b) illustrates the x -intercept method for solving. Identify the equation that is graphed.
- (c) **Writing to Learn** How are the intersection points in (a) related to the x -intercepts in (b)?
- 46. Writing to Learn Revisiting Example 6** Explain why all real numbers x that satisfy $1.324 < x < 1.325$ round to 1.32.

In Exercises 47–56, use a method of your choice to solve the equation.

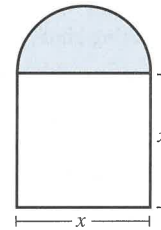
47. $x^2 + x - 2 = 0$ 48. $x^2 - 3x = 12 - 3(x - 2)$
49. $|2x - 1| = 5$ 50. $x + 2 - 2\sqrt{x+3} = 0$
51. $x^3 + 4x^2 - 3x - 2 = 0$ 52. $x^3 - 4x + 2 = 0$
53. $|x^2 + 4x - 1| = 7$ 54. $|x + 5| = |x - 3|$
55. $|0.5x + 3| = x^2 - 4$ 56. $\sqrt{x+7} = -x^2 + 5$
- 57. Group Activity Discriminant of a Quadratic** The radicand $b^2 - 4ac$ in the quadratic formula is called the **discriminant** of the quadratic polynomial $ax^2 + bx + c$ because it can be used to describe the nature of its zeros.
- (a) **Writing to Learn** If $b^2 - 4ac > 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.
- (b) **Writing to Learn** If $b^2 - 4ac = 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.
- (c) **Writing to Learn** If $b^2 - 4ac < 0$, what can you say about the zeros of the quadratic polynomial $ax^2 + bx + c$? Explain your answer.

- 58. Group Activity Discriminant of a Quadratic** Use the information learned in Exercise 57 to create a quadratic polynomial with the following numbers of real zeros. Support your answers graphically.
- (a) Two real zeros
- (b) Exactly one real zero
- (c) No real zeros

- 59. Size of a Soccer Field** Several of the World Cup '94 soccer matches were played in Stanford University's stadium in Menlo Park, California. The field is 30 yd longer than it is wide, and the area of the field is 8800 yd^2 . What are the dimensions of this soccer field?
- 60. Height of a Ladder** John's paint crew knows from experience that its 18-ft ladder is particularly stable when the distance from the ground to the top of the ladder is 5 ft more than the distance from the building to the base of the ladder as shown in the figure. In this position, how far up the building does the ladder reach?



- 61. Finding the Dimensions of a Norman Window** A Norman window has the shape of a square with a semicircle mounted on it. Find the width of the window if the total area of the square and the semicircle is to be 200 ft^2 .



Standardized Test Questions

- 62. True or False** If 2 is an x -intercept of the graph of $y = ax^2 + bx + c$, then 2 is a solution of the equation $ax^2 + bx + c = 0$. Justify your answer.
- 63. True or False** If $2x^2 = 18$, then x must be equal to 3. Justify your answer.

In Exercises 64–67, you may use a graphing calculator to solve these problems.

- 64. Multiple Choice** Which of the following are the solutions of the equation $x(x - 3) = 0$?
- (A) Only $x = 3$ (B) Only $x = -3$
- (C) $x = 0$ and $x = -3$ (D) $x = 0$ and $x = 3$
- (E) There are no solutions.

65. **Multiple Choice** Which of the following replacements for ? make $x^2 - 5x + ?$ a perfect square?

(A) $-\frac{5}{2}$

(B) $\left(-\frac{5}{2}\right)^2$

(C) $(-5)^2$

(D) $\left(-\frac{2}{5}\right)^2$

(E) -6

66. **Multiple Choice** Which of the following are the solutions of the equation $2x^2 - 3x - 1 = 0$?

(A) $\frac{3}{4} \pm \sqrt{17}$

(B) $\frac{3 \pm \sqrt{17}}{4}$

(C) $\frac{3 \pm \sqrt{17}}{2}$

(D) $\frac{-3 \pm \sqrt{17}}{4}$

(E) $\frac{3 \pm 1}{4}$

67. **Multiple Choice** Which of the following are the solutions of the equation $|x - 1| = -3$?

(A) Only $x = 4$

(B) Only $x = -2$

(C) Only $x = 2$

(D) $x = 4$ and $x = -2$

(E) There are no solutions.

Explorations

68. **Deriving the Quadratic Formula** Follow these steps to use completing the square to solve $ax^2 + bx + c = 0$, $a \neq 0$.

- (a) Subtract c from both sides of the original equation and divide both sides of the resulting equation by a to obtain

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

- (b) Add the square of one-half of the coefficient of x in (a) to both sides and simplify to obtain

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

- (c) Extract square roots in (b) and solve for x to obtain the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Extending the Ideas

69. **Finding Number of Solutions** Consider the equation $|x^2 - 4| = c$

- (a) Find a value of c for which this equation has four solutions. (There are many such values.)

- (b) Find a value of c for which this equation has three solutions. (There is only one such value.)

- (c) Find a value of c for which this equation has two solutions. (There are many such values.)

- (d) Find a value of c for which this equation has no solutions. (There are many such values.)

- (e) **Writing to Learn** Are there any other possible numbers of solutions of this equation? Explain.

70. **Sums and Products of Solutions of $ax^2 + bx + c = 0$, $a \neq 0$** Suppose that $b^2 - 4ac > 0$.

- (a) Show that the sum of the two solutions of this equation is $-(b/a)$.

- (b) Show that the product of the two solutions of this equation is c/a .

71. **Exercise 70 Continued** The equation $2x^2 + bx + c = 0$ has two solutions x_1 and x_2 . If $x_1 + x_2 = 5$ and $x_1 \cdot x_2 = 3$, find the two solutions.

P.6

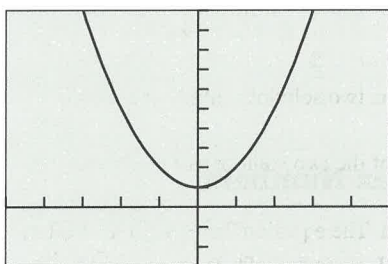
Complex Numbers

What you'll learn about

- Complex Numbers
- Operations with Complex Numbers
- Complex Conjugates and Division
- Complex Solutions of Quadratic Equations

... and why

The zeros of polynomials are complex numbers.



$[-5, 5]$ by $[-3, 10]$

FIGURE P.39 The graph of $f(x) = x^2 + 1$ has no x -intercepts.

HISTORICAL NOTE

René Descartes (1596–1650) coined the term *imaginary* in a time when negative solutions to equations were considered *false*. Carl Friedrich Gauss (1777–1855) gave us the term *complex number* and the symbol i for $\sqrt{-1}$. Today practical applications of complex numbers abound.

Complex Numbers

Figure P.39 shows that the function $f(x) = x^2 + 1$ has no real zeros, so $x^2 + 1 = 0$ has no real-number solutions. To remedy this situation, mathematicians in the 17th century extended the definition of \sqrt{a} to include negative real numbers a . First the number $i = \sqrt{-1}$ is defined as a solution of the equation $i^2 + 1 = 0$ and is the **imaginary unit**. Then for any negative real number $\sqrt{a} = \sqrt{|a|} \cdot i$.

The extended system of numbers, called the *complex numbers*, consists of all real numbers and sums of real numbers and real number multiples of i . The following are all examples of complex numbers:

$$-6, \quad 5i, \quad \sqrt{5}, \quad -7i, \quad \frac{5}{2}i + \frac{2}{3}, \quad -2 + 3i, \quad 5 - 3i, \quad \frac{1}{3} + \frac{4}{5}i.$$

DEFINITION Complex Number

A **complex number** is any number that can be written in the form

$$a + bi,$$

where a and b are real numbers. The real number a is the **real part**, the real number b is the **imaginary part**, and $a + bi$ is the **standard form**.

A real number a is the complex number $a + 0i$, so *all real numbers are also complex numbers*. If $a = 0$ and $b \neq 0$, then $a + bi$ becomes bi , and is an **imaginary number**. For instance, $5i$ and $-7i$ are imaginary numbers.

Two complex numbers are **equal** if and only if their real and imaginary parts are equal. For example,

$$x + yi = 2 + 5i \quad \text{if and only if } x = 2 \text{ and } y = 5.$$

Operations with Complex Numbers

Adding complex numbers is done by adding their real and imaginary parts separately. Subtracting complex numbers is also done using the same parts.

DEFINITION Addition and Subtraction of Complex Numbers

If $a + bi$ and $c + di$ are two complex numbers, then

Sum: $(a + bi) + (c + di) = (a + c) + (b + d)i,$

Difference: $(a + bi) - (c + di) = (a - c) + (b - d)i.$

EXAMPLE 1 Adding and Subtracting Complex Numbers

(a) $(7 - 3i) + (4 + 5i) = (7 + 4) + (-3 + 5)i = 11 + 2i$

(b) $(2 - i) - (8 + 3i) = (2 - 8) + (-1 - 3)i = -6 - 4i$

Now try Exercise 3.

The **additive identity** for the complex numbers is $0 = 0 + 0i$. The **additive inverse** of $a + bi$ is $-(a + bi) = -a - bi$ because

$$(a + bi) + (-a - bi) = 0 + 0i = 0.$$

Many of the properties of real numbers also hold for complex numbers. These include:

- Commutative properties of addition and multiplication,
- Associative properties of addition and multiplication, and
- Distributive properties of multiplication over addition and subtraction.

Using these properties and the fact that $i^2 = -1$, complex numbers can be multiplied by treating them as algebraic expressions.

EXAMPLE 2 Multiplying Complex Numbers

$$\begin{aligned} (2 + 3i) \cdot (5 - i) &= 2(5 - i) + 3i(5 - i) \\ &= 10 - 2i + 15i - 3i^2 \\ &= 10 + 13i - 3(-1) \\ &= 13 + 13i \end{aligned}$$

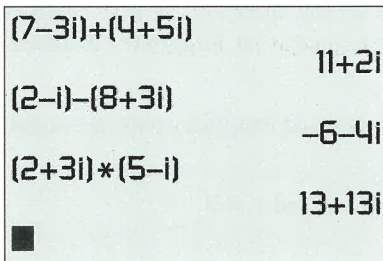
Now try Exercise 9.

FIGURE P.40 Complex number operations on a grapher. (Examples 1 and 2)

We can generalize Example 2 as follows:

$$\begin{aligned} (a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= (ac - bd) + (ad + bc)i \end{aligned}$$

Many graphers can perform basic calculations on complex numbers. Figure P.40 shows how the operations of Examples 1 and 2 look on some graphers.

We compute positive integer powers of complex numbers by treating them as algebraic expressions.

EXAMPLE 3 Raising a Complex Number to a Power

If $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, find z^2 and z^3 .

continued

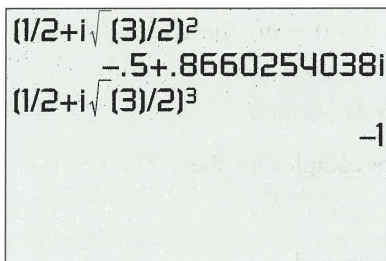


FIGURE P.41 The square and cube of a complex number. (Example 3)

SOLUTION

$$\begin{aligned}
 z^2 &= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \frac{1}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2 \\
 &= \frac{1}{4} + \frac{2\sqrt{3}}{4}i + \frac{3}{4}(-1) \\
 &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i \\
 z^3 &= z^2 \cdot z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= -\frac{1}{4} - \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2 \\
 &= -\frac{1}{4} + 0i + \frac{3}{4}(-1) \\
 &= -1
 \end{aligned}$$

Figure P.41 supports these results numerically.

Now try Exercise 27.

Example 3 demonstrates that $1/2 + (\sqrt{3}/2)i$ is a cube root of -1 and a solution of $x^3 + 1 = 0$. In Section 2.5 complex zeros of polynomial functions will be explored in depth.

Complex Conjugates and Division

The product of the complex numbers $a + bi$ and $a - bi$ is a positive real number:

$$(a + bi) \cdot (a - bi) = a^2 - (bi)^2 = a^2 + b^2.$$

We introduce the following definition to describe this special relationship.

DEFINITION Complex Conjugate

The **complex conjugate** of the complex number $z = a + bi$ is

$$\bar{z} = \overline{a + bi} = a - bi.$$

The **multiplicative identity** for the complex numbers is $1 = 1 + 0i$. The **multiplicative inverse**, or **reciprocal**, of $z = a + bi$ is

$$z^{-1} = \frac{1}{z} = \frac{1}{a + bi} = \frac{1}{a + bi} \cdot \frac{a - bi}{a - bi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}i.$$

In general, a quotient of two complex numbers, written in fraction form, can be simplified as we just simplified $1/z$ —by multiplying the numerator and denominator of the fraction by the complex conjugate of the denominator.

EXAMPLE 4 Dividing Complex Numbers

Write the complex number in standard form.

(a) $\frac{2}{3-i}$

(b) $\frac{5+i}{2-3i}$

SOLUTION Multiply the numerator and denominator by the complex conjugate of the denominator.

$$\begin{aligned} \text{(a)} \quad \frac{2}{3-i} &= \frac{2}{3-i} \cdot \frac{3+i}{3+i} & \text{(b)} \quad \frac{5+i}{2-3i} &= \frac{5+i}{2-3i} \cdot \frac{2+3i}{2+3i} \\ &= \frac{6+2i}{3^2+1^2} & &= \frac{10+15i+2i+3i^2}{2^2+3^2} \\ &= \frac{6}{10} + \frac{2}{10}i & &= \frac{7+17i}{13} \\ &= \frac{3}{5} + \frac{1}{5}i & &= \frac{7}{13} + \frac{17}{13}i \end{aligned}$$

Now try Exercise 33.

Complex Solutions of Quadratic Equations

Recall that the solutions of the quadratic equation $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$, are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The radicand $b^2 - 4ac$ is the **discriminant**, and tells us whether the solutions are real numbers. In particular, if $b^2 - 4ac < 0$, the solutions involve the square root of a negative number and so lead to complex-number solutions. In all, there are three cases, which we now summarize:

Discriminant of a Quadratic Equation

For a quadratic equation $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$,

- If $b^2 - 4ac > 0$, there are two distinct real solutions.
- If $b^2 - 4ac = 0$, there is one repeated real solution.
- If $b^2 - 4ac < 0$, there is a complex conjugate pair of solutions.

EXAMPLE 5 Solving a Quadratic EquationSolve $x^2 + x + 1 = 0$.**SOLUTION****Solve Algebraically**Using the quadratic formula with $a = b = c = 1$, we obtain

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

So the solutions are $-1/2 + (\sqrt{3}/2)i$ and $-1/2 - (\sqrt{3}/2)i$, a complex conjugate pair.**Confirm Numerically**Substituting $-1/2 + (\sqrt{3}/2)i$ into the original equation, we obtain

$$\begin{aligned} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + 1 \\ = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + 1 = 0 \end{aligned}$$

By a similar computation we can confirm the second solution.

Now try Exercise 41.**QUICK REVIEW P.6**

In Exercises 1–4, add or subtract, and simplify.

- $(2x + 3) + (-x + 6)$
- $(3y - x) + (2x - y)$
- $(2a + 4d) - (a + 2d)$
- $(6z - 1) - (z + 3)$

In Exercises 5–10, multiply and simplify.

- $(x - 3)(x + 2)$
- $(2x - 1)(x + 3)$
- $(x - \sqrt{2})(x + \sqrt{2})$
- $(x + 2\sqrt{3})(x - 2\sqrt{3})$
- $[x - (1 + \sqrt{2})][x - (1 - \sqrt{2})]$
- $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$

SECTION P.6 EXERCISESIn Exercises 1–8, write the sum or difference in the standard form $a + bi$.

- $(2 - 3i) + (6 + 5i)$
- $(2 - 3i) + (3 - 4i)$
- $(7 - 3i) + (6 - i)$
- $(2 + i) - (9i - 3)$
- $(2 - i) + (3 - \sqrt{-3})$
- $(\sqrt{5} - 3i) + (-2 + \sqrt{-9})$
- $(i^2 + 3) - (7 + i^3)$
- $(\sqrt{7} + i^2) - (6 - \sqrt{-81})$

In Exercises 9–16, write the product in standard form.

- $(2 + 3i)(2 - i)$
- $(2 - i)(1 + 3i)$
- $(1 - 4i)(3 - 2i)$
- $(5i - 3)(2i + 1)$
- $(7i - 3)(2 + 6i)$
- $(\sqrt{-4} + i)(6 - 5i)$
- $(-3 - 4i)(1 + 2i)$
- $(\sqrt{-2} + 2i)(6 + 5i)$

In Exercises 17–20, write the expression in the form bi , where b is a real number.

- $\sqrt{-16}$
- $\sqrt{-25}$

19. $\sqrt{-3}$

20. $\sqrt{-5}$

In Exercises 21–24, find the real numbers x and y that make the equation true.

21. $2 + 3i = x + yi$

22. $3 + yi = x - 7i$

23. $(5 - 2i) - 7 = x - (3 + yi)$

24. $(x + 6i) = (3 - i) + (4 - 2yi)$

In Exercises 25–28, write the complex number in standard form.

25. $(3 + 2i)^2$

26. $(1 - i)^3$

27. $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4$

28. $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$

In Exercises 29–32, find the product of the complex number and its conjugate.

29. $2 - 3i$

30. $5 - 6i$

31. $-3 + 4i$

32. $-1 - \sqrt{2}i$

In Exercises 33–40, write the expression in standard form.

33. $\frac{1}{2 + i}$

34. $\frac{i}{2 - i}$

35. $\frac{2 + i}{2 - i}$

36. $\frac{2 + i}{3i}$

37. $\frac{(2 + i)^2(-i)}{1 + i}$

38. $\frac{(2 - i)(1 + 2i)}{5 + 2i}$

39. $\frac{(1 - i)(2 - i)}{1 - 2i}$

40. $\frac{(1 - \sqrt{2}i)(1 + i)}{(1 + \sqrt{2}i)}$

In Exercises 41–44, solve the equation.

41. $x^2 + 2x + 5 = 0$

42. $3x^2 + x + 2 = 0$

43. $4x^2 - 6x + 5 = x + 1$

44. $x^2 + x + 11 = 5x - 8$

Standardized Test Questions

45. **True or False** There are no complex numbers z satisfying $z = -\bar{z}$. Justify your answer.

46. **True or False** For the complex number i , $i + i^2 + i^3 + i^4 = 0$. Justify your answer.

In Exercises 47–50, solve the problem without using a calculator.

47. **Multiple Choice** Which of the following is the standard form for the product $(2 + 3i)(2 - 3i)$?

- (A) $-5 + 12i$ (B) $4 - 9i$ (C) $13 - 3i$ (D) -5 (E) 13

48. **Multiple Choice** Which of the following is the standard form for the quotient $\frac{1}{i}$?

- (A) 1 (B) -1 (C) i (D) $-i$ (E) $-1 + i$

49. **Multiple Choice** Assume that $2 - 3i$ is a solution of $ax^2 + bx + c = 0$, where a, b, c are real numbers. Which of the following is also a solution of the equation?

- (A) $2 + 3i$ (B) $-2 - 3i$ (C) $-2 + 3i$

- (D) $3 + 2i$ (E) $\frac{1}{2 - 3i}$

50. **Multiple Choice** Which of the following is the standard form for the power $(1 - i)^3$?

- (A) $-4i$ (B) $-2 + 2i$ (C) $-2 - 2i$ (D) $2 + 2i$ (E) $2 - 2i$

Explorations

51. **Group Activity The Powers of i**

(a) Simplify the complex numbers i, i^2, \dots, i^8 by evaluating each one.

(b) Simplify the complex numbers $i^{-1}, i^{-2}, \dots, i^{-8}$ by evaluating each one.

(c) Evaluate i^0 .

(d) **Writing to Learn** Discuss your results from (a)–(c) with the members of your group, and write a summary statement about the integer powers of i .

52. **Writing to Learn** Describe the nature of the graph of $f(x) = ax^2 + bx + c$ when a, b , and c are real numbers and the equation $ax^2 + bx + c = 0$ has nonreal complex solutions.

Extending the Ideas

53. Prove that the difference between a complex number and its conjugate is a complex number whose real part is 0.

54. Prove that the product of a complex number and its complex conjugate is a complex number whose imaginary part is zero.

55. Prove that the complex conjugate of a product of two complex numbers is the product of their complex conjugates.

56. Prove that the complex conjugate of a sum of two complex numbers is the sum of their complex conjugates.

57. **Writing to Learn** Explain why $-i$ is a solution of $x^2 - ix + 2 = 0$ but i is not.

P.7

Solving Inequalities Algebraically and Graphically

What you'll learn about

- Solving Absolute Value Inequalities
- Solving Quadratic Inequalities
- Approximating Solutions to Inequalities
- Projectile Motion

... and why

These techniques are involved in using a graphing utility to solve inequalities in this textbook.

Solving Absolute Value Inequalities

The methods for solving inequalities parallel the methods for solving equations. Here are two basic rules we apply to solve absolute value inequalities.

Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let a be a real number with $a \geq 0$.

1. If $|u| < a$, then u is in the interval $(-a, a)$. That is,

$$|u| < a \quad \text{if and only if} \quad -a < u < a.$$

2. If $|u| > a$, then u is in the interval $(-\infty, -a)$ or (a, ∞) , that is,

$$|u| > a \quad \text{if and only if} \quad u < -a \quad \text{or} \quad u > a.$$

The inequalities $<$ and $>$ can be replaced with \leq and \geq , respectively. See Figure P.42.

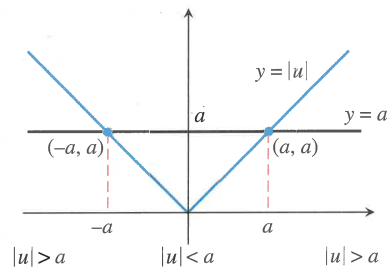


FIGURE P.42 The solution of $|u| < a$ is represented by the portion of the number line where the graph of $y = |u|$ is below the graph of $y = a$. The solution of $|u| > a$ is represented by the portion of the number line where the graph of $y = |u|$ is above the graph of $y = a$.

EXAMPLE 1 Solving an Absolute Value Inequality

Solve $|x - 4| < 8$.

SOLUTION

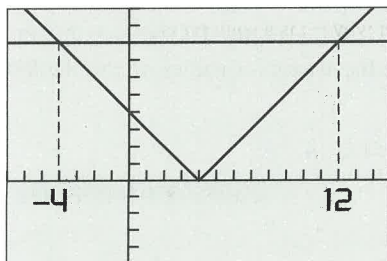
$$|x - 4| < 8 \quad \text{Original inequality}$$

$$-8 < x - 4 < 8 \quad \text{Equivalent double inequality}$$

$$-4 < x < 12 \quad \text{Add 4.}$$

As an interval the solution is $(-4, 12)$.

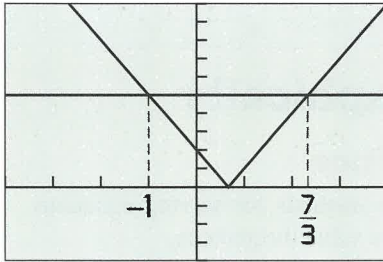
Figure P.43 shows that points on the graph of $y = |x - 4|$ are below the points on the graph of $y = 8$ for values of x between -4 and 12 .



$[-7, 15]$ by $[-5, 10]$

FIGURE P.43 The graphs of $y = |x - 4|$ and $y = 8$. (Example 1)

Now try Exercise 3.



$[-4, 4]$ by $[-4, 10]$

FIGURE P.44 The graphs of $y = |3x - 2|$ and $y = 5$. (Example 2)

UNION OF TWO SETS

The **union of two sets A and B** , denoted by $A \cup B$, is the set of all objects that belong to A or B or both.

EXAMPLE 2 Solving Another Absolute Value Inequality

Solve $|3x - 2| \geq 5$.

SOLUTION The solution of this absolute value inequality consists of the solutions of both of these inequalities.

$$3x - 2 \leq -5 \quad \text{or} \quad 3x - 2 \geq 5$$

$$3x \leq -3 \quad \text{or} \quad 3x \geq 7 \quad \text{Add 2.}$$

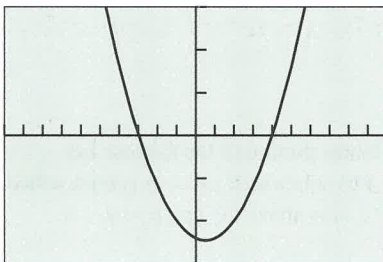
$$x \leq -1 \quad \text{or} \quad x \geq \frac{7}{3} \quad \text{Divide by 3.}$$

The solution consists of all numbers that are in either one of the two intervals $(-\infty, -1]$ or $[7/3, \infty)$, which may be written as $(-\infty, -1] \cup [7/3, \infty)$. The notation “ \cup ” is read as “union.”

Figure P.44 shows that points on the graph of $y = |3x - 2|$ are above or on the points on the graph of $y = 5$ for values of x to the left of and including -1 and to the right of and including $7/3$. **Now try Exercise 7.**

Solving Quadratic Inequalities

To solve a quadratic inequality such as $x^2 - x - 12 > 0$ we begin by solving the corresponding quadratic equation $x^2 - x - 12 = 0$. Then we determine the values of x for which the graph of $y = x^2 - x - 12$ lies above the x -axis.



$[-10, 10]$ by $[-15, 15]$

FIGURE P.45 The graph of $y = x^2 - x - 12$ appears to cross the x -axis at $x = -3$ and $x = 4$. (Example 3)

EXAMPLE 3 Solving a Quadratic Inequality

Solve $x^2 - x - 12 > 0$.

SOLUTION First we solve the corresponding equation $x^2 - x - 12 = 0$.

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0 \quad \text{Factor.}$$

$$x - 4 = 0 \quad \text{or} \quad x + 3 = 0 \quad ab = 0 \Rightarrow a = 0 \text{ or } b = 0$$

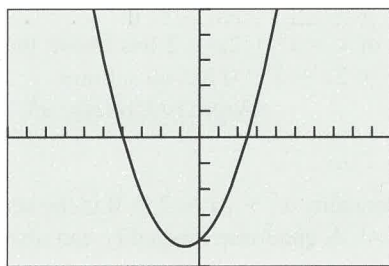
$$x = 4 \quad \text{or} \quad x = -3 \quad \text{Solve for } x.$$

The solutions of the corresponding quadratic equation are -3 and 4 , and they are not solutions of the original inequality because $0 > 0$ is false. Figure P.45 shows that the points on the graph of $y = x^2 - x - 12$ are above the x -axis for values of x to the left of -3 and to the right of 4 .

The solution of the original inequality is $(-\infty, -3) \cup (4, \infty)$.

Now try Exercise 11.

In Example 4, the quadratic inequality involves the symbol \leq . In this case, the solutions of the corresponding quadratic equation are also solutions of the inequality.



$[-10, 10]$ by $[-25, 25]$

FIGURE P.46 The graph of $y = 2x^2 + 3x - 20$ appears to be below the x -axis for $-4 < x < 2.5$. (Example 4)

EXAMPLE 4 Solving Another Quadratic Inequality

Solve $2x^2 + 3x \leq 20$.

SOLUTION First we subtract 20 from both sides of the inequality to obtain $2x^2 + 3x - 20 \leq 0$. Next, we solve the corresponding quadratic equation $2x^2 + 3x - 20 = 0$.

$$\begin{aligned} 2x^2 + 3x - 20 &= 0 \\ (x + 4)(2x - 5) &= 0 && \text{Factor.} \\ x + 4 = 0 &\quad \text{or} \quad 2x - 5 = 0 && ab = 0 \Rightarrow a = 0 \text{ or } b = 0 \\ x = -4 &\quad \text{or} \quad x = \frac{5}{2} && \text{Solve for } x. \end{aligned}$$

The solutions of the corresponding quadratic equation are -4 and $5/2 = 2.5$. You can check that they are also solutions of the inequality.

Figure P.46 shows that the points on the graph of $y = 2x^2 + 3x - 20$ are below the x -axis for values of x between -4 and 2.5 . The solution of the original inequality is $[-4, 2.5]$. We use square brackets because the numbers -4 and 2.5 are also solutions of the inequality.

Now try Exercise 9.

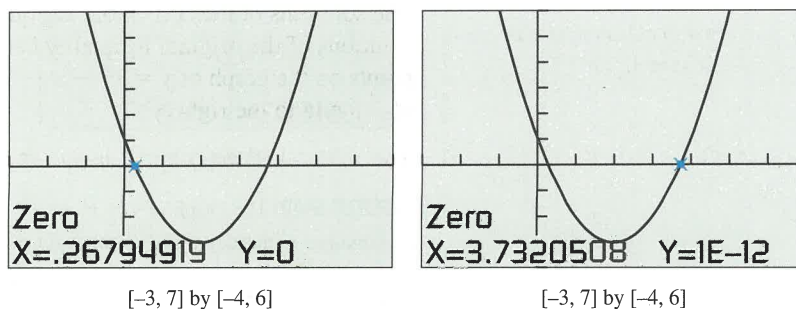
In Examples 3 and 4 the corresponding quadratic equation factored. If this doesn't happen we will need to approximate the zeros of the quadratic equation if it has any. Then we use our accuracy agreement from Section P.5 and write the endpoints of any intervals accurate to two decimal places as illustrated in Example 5.

EXAMPLE 5 Solving a Quadratic Inequality Graphically

Solve $x^2 - 4x + 1 \geq 0$ graphically.

SOLUTION We can use the graph of $y = x^2 - 4x + 1$ in Figure P.47 to determine that the solutions of the equation $x^2 - 4x + 1 = 0$ are about 0.27 and 3.73 . Thus, the solution of the original inequality is $(-\infty, 0.27] \cup [3.73, \infty)$. We use square brackets because the zeros of the quadratic equation are solutions of the inequality even though we only have approximations to their values.

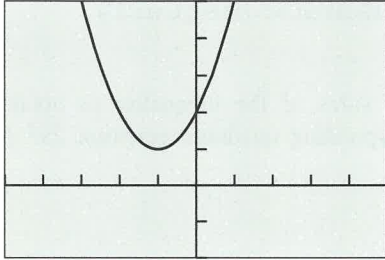
Now try Exercise 21.



$[-3, 7]$ by $[-4, 6]$

$[-3, 7]$ by $[-4, 6]$

FIGURE P.47 This figure suggests that $y = x^2 - 4x + 1$ is zero for $x \approx 0.27$ and $x \approx 3.73$. (Example 5)



$[-5, 5]$ by $[-2, 5]$

FIGURE P.48 The values of $y = x^2 + 2x + 2$ are never negative. (Example 6)

EXAMPLE 6 Showing that a Quadratic Inequality has no Solution

Solve $x^2 + 2x + 2 < 0$.

SOLUTION Figure P.48 shows that the graph of $y = x^2 + 2x + 2$ lies above the x -axis for all values for x . Thus, the inequality $x^2 + 2x + 2 < 0$ has *no* solution.

Now try Exercise 25.

Figure P.48 also shows that the solutions of the inequality $x^2 + 2x + 2 > 0$ is the set of all real numbers or, in interval notation, $(-\infty, \infty)$. A quadratic inequality can also have exactly one solution (see Exercise 31).

Approximating Solutions to Inequalities

To solve the inequality in Example 7 we approximate the zeros of the corresponding graph. Then we determine the values of x for which the corresponding graph is above or on the x -axis.

EXAMPLE 7 Solving a Cubic Inequality

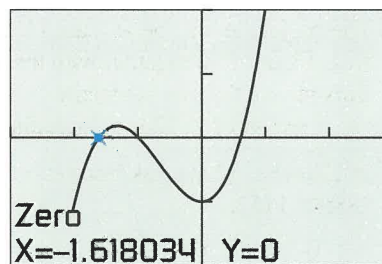
Solve $x^3 + 2x^2 - 1 \geq 0$ graphically.

SOLUTION

We can use the graph of $y = x^3 + 2x^2 - 1$ in Figure P.49 to show that the solutions of the corresponding equation $x^3 + 2x^2 - 1 = 0$ are approximately -1.62 , -1 , and 0.62 . The points on the graph of $y = x^3 + 2x^2 - 1$ are above the x -axis for values of x between -1.62 and -1 , and for values of x to the right of 0.62 .

The solution of the inequality is $[-1.62, -1] \cup [0.62, \infty)$. We use square brackets because the zeros of $y = x^3 + 2x^2 - 1$ are also solutions of the inequality.

Now try Exercise 27.



$[-3, 3]$ by $[-2, 2]$

FIGURE P.49 The graph of $y = x^3 + 2x^2 - 1$ appears to be above the x -axis between the two negative x -intercepts and to the right of the positive x -intercept. (Example 7)

Projectile Motion

The movement of an object that is propelled vertically, but then subject only to the force of gravity, is an example of **projectile motion**.

Projectile Motion

Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is

$$s = -16t^2 + v_0t + s_0.$$

EXAMPLE 8 Finding Height of a Projectile

A projectile is launched straight up from ground level with an initial velocity of 288 ft/sec.

- (a) When will the projectile's height above ground be 1152 ft?
 (b) When will the projectile's height above ground be at least 1152 ft?

SOLUTION

Here $s_0 = 0$ and $v_0 = 288$. So, the projectile's height is $s = -16t^2 + 288t$.

- (a) We need to determine when $s = 1152$.

$$s = -16t^2 + 288t$$

$$1152 = -16t^2 + 288t$$

Substitute $s = 1152$.

$$16t^2 - 288t + 1152 = 0$$

Add $16t^2 - 288t$.

$$t^2 - 18t + 72 = 0$$

Divide by 16.

$$(t - 6)(t - 12) = 0$$

Factor.

$$t = 6 \quad \text{or} \quad t = 12$$

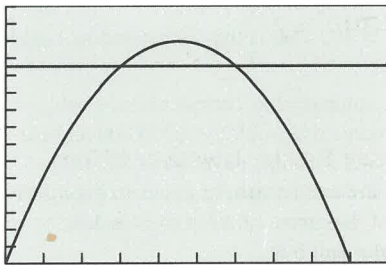
Solve for t .

The projectile is 1152 ft above ground twice; the first time at $t = 6$ sec on the way up, and the second time at $t = 12$ sec on the way down (Figure P.50).

- (b) The projectile will be at least 1152 ft above ground when $s \geq 1152$. We can see from Figure P.50 together with the algebraic work in (a) that the solution is $[6, 12]$. This means that the projectile is at least 1152 ft above ground for times between $t = 6$ sec and $t = 12$ sec, including 6 and 12 sec.

In Exercise 32 we ask you to use algebra to solve the inequality $s = -16t^2 + 288t \geq 1152$.

Now try Exercise 33.



$[0, 20]$ by $[0, 1500]$

FIGURE P.50 The graphs of $s = -16t^2 + 288t$ and $s = 1152$. We know from Example 8a that the two graphs intersect at $(6, 1152)$ and $(12, 1152)$.

QUICK REVIEW P.7

In Exercises 1–3, solve for x .

1. $-7 < 2x - 3 < 7$ 2. $5x - 2 \geq 7x + 4$
 3. $|x + 2| = 3$

In Exercises 4–6, factor the expression completely.

4. $4x^2 - 9$ 5. $x^3 - 4x$
 6. $9x^2 - 16y^2$

In Exercises 7 and 8, reduce the fraction to lowest terms.

7. $\frac{z^2 - 25}{z^2 - 5z}$ 8. $\frac{x^2 + 2x - 35}{x^2 - 10x + 25}$

In Exercises 9 and 10, add the fractions and simplify.

9. $\frac{x}{x-1} + \frac{x+1}{3x-4}$ 10. $\frac{2x-1}{x^2-x-2} + \frac{x-3}{x^2-3x+2}$

SECTION P.7 EXERCISES

In Exercises 1–8, solve the inequality algebraically. Write the solution in interval notation and draw its number line graph.

1. $|x + 4| \geq 5$
2. $|2x - 1| > 3.6$
3. $|x - 3| < 2$
4. $|x + 3| \leq 5$
5. $|4 - 3x| - 2 < 4$
6. $|3 - 2x| + 2 > 5$
7. $\left| \frac{x + 2}{3} \right| \geq 3$
8. $\left| \frac{x - 5}{4} \right| \leq 6$

In Exercises 9–16, solve the inequality. Use algebra to solve the corresponding equation.

9. $2x^2 + 17x + 21 \leq 0$
10. $6x^2 - 13x + 6 \geq 0$
11. $2x^2 + 7x > 15$
12. $4x^2 + 2 < 9x$
13. $2 - 5x - 3x^2 < 0$
14. $21 + 4x - x^2 > 0$
15. $x^3 - x \geq 0$
16. $x^3 - x^2 - 30x \leq 0$

In Exercises 17–26, solve the inequality graphically.

17. $x^2 - 4x < 1$
18. $12x^2 - 25x + 12 \geq 0$
19. $6x^2 - 5x - 4 > 0$
20. $4x^2 - 1 \leq 0$
21. $9x^2 + 12x - 1 \geq 0$
22. $4x^2 - 12x + 7 < 0$
23. $4x^2 + 1 > 4x$
24. $x^2 + 9 \leq 6x$
25. $x^2 - 8x + 16 < 0$
26. $9x^2 + 12x + 4 \geq 0$

In Exercises 27–30, solve the cubic inequality graphically.

27. $3x^3 - 12x + 2 \geq 0$
28. $8x - 2x^3 - 1 < 0$
29. $2x^3 + 2x > 5$
30. $4 \leq 2x^3 + 8x$

31. **Group Activity** Give an example of a quadratic inequality with the indicated solution.

- | | |
|--------------------------------------|-------------------------------------|
| (a) All real numbers | (b) No solution |
| (c) Exactly one solution | (d) $[-2, 5]$ |
| (e) $(-\infty, -1) \cup (4, \infty)$ | (f) $(-\infty, 0] \cup [4, \infty)$ |

32. **Revisiting Example 8** Solve the inequality $-16t^2 + 288t \geq 1152$ algebraically and compare your answer with the result obtained in Example 10.

33. **Projectile Motion** A projectile is launched straight up from ground level with an initial velocity of 256 ft/sec.

- (a) When will the projectile's height above ground be 768 ft?
- (b) When will the projectile's height above ground be at least 768 ft?
- (c) When will the projectile's height above ground be less than or equal to 768 ft?

34. **Projectile Motion** A projectile is launched straight up from ground level with an initial velocity of 272 ft/sec.

- (a) When will the projectile's height above ground be 960 ft?

(b) When will the projectile's height above ground be more than 960 ft?

(c) When will the projectile's height above ground be less than or equal to 960 ft?

35. **Writing to Learn** Explain the role of equation solving in the process of solving an inequality. Give an example.

36. **Travel Planning** Barb wants to drive to a city 105 mi from her home in no more than 2 h. What is the lowest average speed she must maintain on the drive?

37. **Connecting Algebra and Geometry** Consider the collection of all rectangles that have length 2 in. less than twice their width.

(a) Find the possible widths (in inches) of these rectangles if their perimeters are less than 200 in.

(b) Find the possible widths (in inches) of these rectangles if their areas are less than or equal to 1200 in.².

38. **Boyle's Law** For a certain gas, $P = 400/V$, where P is pressure and V is volume. If $20 \leq V \leq 40$, what is the corresponding range for P ?

39. **Cash-Flow Planning** A company has current assets (cash, property, inventory, and accounts receivable) of \$200,000 and current liabilities (taxes, loans, and accounts payable) of \$50,000. How much can it borrow if it wants its ratio of assets to liabilities to be no less than 2? Assume the amount borrowed is added to both current assets and current liabilities.

Standardized Test Questions

40. **True or False** The absolute value inequality $|x - a| < b$, where a and b are real numbers, always has at least one solution. Justify your answer.

41. **True or False** Every real number is a solution of the absolute value inequality $|x - a| \geq 0$, where a is a real number. Justify your answer.

In Exercises 42–45, solve these problems without using a calculator.

42. **Multiple Choice** Which of the following is the solution to $|x - 2| < 3$?

- | | |
|-------------------------|--------------------------------------|
| (A) $x = -1$ or $x = 5$ | (B) $[-1, 5)$ |
| (C) $[-1, 5]$ | (D) $(-\infty, -1) \cup (5, \infty)$ |
| (E) $(-1, 5)$ | |

43. **Multiple Choice** Which of the following is the solution to $x^2 - 2x + 2 \geq 0$?

- | | |
|-------------------------------------|-------------------------------------|
| (A) $[0, 2]$ | (B) $(-\infty, 0) \cup (2, \infty)$ |
| (C) $(-\infty, 0] \cup [2, \infty)$ | (D) All real numbers |
| (E) There is no solution. | |

44. Multiple Choice Which of the following is the solution to $x^2 > x$?

- (A) $(-\infty, 0) \cup (1, \infty)$ (B) $(-\infty, 0] \cup [1, \infty)$
 (C) $(1, \infty)$ (D) $(0, \infty)$
 (E) There is no solution.

45. Multiple Choice Which of the following is the solution to $x^2 \leq 1$?

- (A) $(-\infty, 1]$ (B) $(-1, 1)$
 (C) $[1, \infty)$ (D) $[-1, 1]$
 (E) There is no solution.

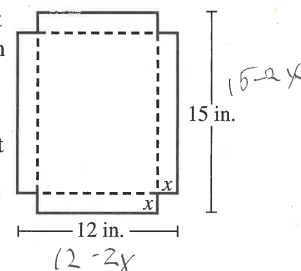
Explorations

46. Constructing a Box with No Top An open box is formed by cutting squares from the corners of a regular piece of cardboard (see figure) and folding up the flaps.

(a) What size corner squares should be cut to yield a box with a volume of 125 in.^3 ?

(b) What size corner squares should be cut to yield a box with a volume more than 125 in.^3 ?

(c) What size corner squares should be cut to yield a box with a volume of at most 125 in.^3 ?



Extending the Ideas

In Exercises 47 and 48, use a combination of algebraic and graphical techniques to solve the inequalities.

47. $|2x^2 + 7x - 15| < 10$ **48.** $|2x^2 + 3x - 20| \geq 10$

CHAPTER P Key Ideas

PROPERTIES, THEOREMS, AND FORMULAS

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CHAPTER P Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1 and 2, find the endpoints and state whether the interval is bounded or unbounded.

1. $[0, 5]$ 2. $(2, \infty)$

3. **Distributive Property** Use the distributive property to write the expanded form of $2(x^2 - x)$.

4. **Distributive Property** Use the distributive property to write the factored form of $2x^3 + 4x^2$.

In Exercises 5 and 6, simplify the expression. Assume that denominators are not zero.

5. $\frac{(uv^2)^3}{v^2u^3}$

6. $(3x^2y^3)^{-2}$

In Exercises 7 and 8, write the number in scientific notation.

7. The mean distance from Pluto to the sun is about 3,680,000,000 miles.
8. The diameter of a red blood corpuscle is about 0.000007 meter.

In Exercises 9 and 10, write the number in decimal form.

9. Our solar system is about 5×10^9 years old.
10. The mass of an electron is about 9.1094×10^{-28} g (gram).
11. The data in Table P.9 gives the spending on collegiate student financial aid from several sources. Write the spending amount in dollars by the given source in scientific notation.



Table P.9 Student Financial Aid

Spending Source	Amount (dollars)
Federal Aid	50,711,000,000
State Grant Programs	4,630,000,000
State-sponsored Loans	500,000,000
Private-sector Loans	3,995,000,000
Institution and other Grants	14,497,000,000

Source: The College Board, as reported in *The Chronicle of Higher Education, Almanac, 2002-3, August 30, 2002.*

- (a) Federal Aid
(b) State Grant Programs
(c) State-sponsored Loans
(d) Private-sector Loans
(e) Institution and other Grants

12. **Decimal Form** Find the decimal form for $-5/11$. State whether it repeats or terminates.

In Exercises 13 and 14, find (a) the distance between the points and (b) the midpoint of the line segment determined by the points.

13. -5 and 14

14. $(-4, 3)$ and $(5, -1)$

In Exercises 15 and 16, show that the figure determined by the points is the indicated type.

15. Right triangle: $(-2, 1)$, $(3, 11)$, $(7, 9)$

16. Equilateral triangle: $(0, 1)$, $(4, 1)$, $(2, 1 - 2\sqrt{3})$

In Exercises 17 and 18, find the standard form equation for the circle,

17. Center $(0, 0)$, radius 2

18. Center $(5, -3)$, radius 4

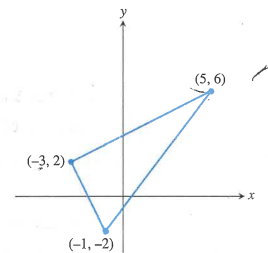
In Exercises 19 and 20, find the center and radius of the circle.

19. $(x + 5)^2 + (y + 4)^2 = 9$

20. $x^2 + y^2 = 1$

21. (a) Find the length of the sides of the triangle in the figure.

(b) **Writing to Learn** Show that the triangle is a right triangle.



22. **Distance and Absolute Value** Use absolute value notation to write the statement that the distance between z and -3 is less than or equal to 1.

23. **Finding a Line Segment with Given Midpoint** Let $(3, 5)$ be the midpoint of the line segment with endpoints $(-1, 1)$ and (a, b) . Determine a and b .

24. **Finding Slope** Find the slope of the line through the points $(-1, -2)$ and $(4, -5)$.

25. **Finding Point-Slope Form Equation** Find an equation in point-slope form for the line through the point $(2, -1)$ with slope $m = -2/3$.

26. Find an equation of the line through the points $(-5, 4)$ and $(2, -5)$ in the general form $Ax + By + C = 0$.

In Exercises 27-32, find an equation in slope-intercept form for the line.

27. The line through $(3, -2)$ with slope $m = 4/5$

28. The line through the points $(-1, -4)$ and $(3, 2)$

29. The line through $(-2, 4)$ with slope $m = 0$

30. The line $3x - 4y = 7$

31. The line through $(2, -3)$ and parallel to the line $2x + 5y = 3$.
32. The line through $(2, -3)$ and perpendicular to the line $2x + 5y = 3$.

33. **SAT Math Scores** The SAT scores are measured on an 800-point scale. The data in Table P.10 shows the average SAT math score for several years.



Table P.10 Average SAT Math Scores

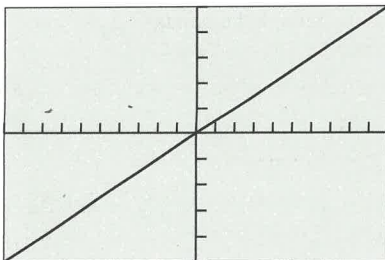
Year	SAT Math Score
1995	506
1997	511
1998	512
1999	511
2000	514
2001	514
2002	516
2003	519
2004	518

Source: *The World Almanac and Book of Facts, The New York Times June, 2005.*

- (a) Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. Draw a scatter plot of the data.
- (b) Use the 1995 and 2000 data to write a linear equation for the average SAT math score y in terms of the year x . Superimpose the graph of the linear equation on the scatter plot in (a).
- (c) Use the equation in (b) to estimate the average SAT math score in 1996. Compare with the actual value of 508.
- (d) Use the equation in (b) to predict the average SAT math score in 2006.
34. Consider the point $(-6, 3)$ and Line $L: 4x - 3y = 5$. Write an equation (a) for the line passing through this point and parallel to L , and (b) for the line passing through this point and perpendicular to L . Support your work graphically.

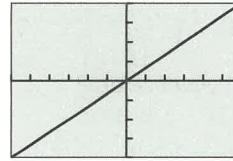
In Exercises 35 and 36, assume that each graph contains the origin and the upper right-hand corner of the viewing window.

35. Find the slope of the line in the figure.



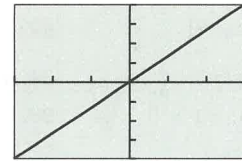
$[-10, 10]$ by $[-25, 25]$

36. **Writing to Learn** Which line has the greater slope? Explain.



$[-6, 6]$ by $[-4, 4]$

(a)



$[-15, 15]$ by $[-12, 12]$

(b)

In Exercises 37–50, solve the equation algebraically.

37. $3x - 4 = 6x + 5$
38. $\frac{x-2}{3} + \frac{x+5}{2} = \frac{1}{3}$
39. $2(5-2y) - 3(1-y) = y + 1$
40. $3(3x-1)^2 = 21$
41. $x^2 - 4x - 3 = 0$
42. $16x^2 - 24x + 7 = 0$
43. $6x^2 + 7x = 3$
44. $2x^2 + 8x = 0$
45. $x(2x+5) = 4(x+7)$
46. $|4x+1| = 3$
47. $4x^2 - 20x + 25 = 0$
48. $-9x^2 + 12x - 4 = 0$
49. $x^2 = 3x$
50. $4x^2 - 4x + 2 = 0$
51. $x^2 - 6x + 13 = 0$
52. $x^2 - 2x + 4 = 0$

53. **Completing the Square** Use completing the square to solve the equation $2x^2 - 3x - 1 = 0$.

54. **Quadratic Formula** Use the quadratic formula to solve the equation $3x^2 + 4x - 1 = 0$.

In Exercises 55–58, solve the equation graphically.

55. $3x^3 - 19x^2 - 14x = 0$
56. $x^3 + 2x^2 - 4x - 8 = 0$
57. $x^3 - 2x^2 - 2 = 0$
58. $|2x - 1| = 4 - x^2$

In Exercises 59 and 60, solve the inequality and draw a number line graph of the solution.

59. $-2 < x + 4 \leq 7$
60. $5x + 1 \geq 2x - 4$

In Exercises 61–72, solve the inequality.

$$61. \frac{3x - 5}{4} \leq -1$$

$$63. |3x + 4| \geq 2$$

$$65. 2x^2 - 2x - 1 > 0$$

$$67. x^3 - 9x \leq 3$$

$$69. \left| \frac{x + 7}{5} \right| > 2$$

$$71. 4x^2 + 12x + 9 \geq 0$$

$$62. |2x - 5| < 7$$

$$64. 4x^2 + 3x > 10$$

$$66. 9x^2 - 12x - 1 \leq 0$$

$$68. 4x^3 - 9x + 2 > 0$$

$$70. 2x^2 + 3x - 35 < 0$$

$$72. x^2 - 6x + 9 < 0$$

In Exercises 73–80, perform the indicated operation, and write the result in the standard form $a + bi$.

$$73. (3 - 2i) + (-2 + 5i)$$

$$75. (1 + 2i)(3 - 2i)$$

$$77. (1 + 2i)^2(1 - 2i)^2$$

$$79. \sqrt{-16}$$

$$74. (5 - 7i) - (3 - 2i)$$

$$76. (1 + i)^3$$

$$78. i^{29}$$

$$80. \frac{2 + 3i}{1 - 5i}$$

81. Projectile Motion A projectile is launched straight up from ground level with an initial velocity of 320 ft/sec.

(a) When will the projectile's height above ground be 1538 ft?

(b) When will the projectile's height above ground be at most 1538 ft?

(c) When will the projectile's height above ground be greater than or equal to 1538 ft?

82. Navigation A commercial jet airplane climbs at takeoff with slope $m = 4/9$. How far in the horizontal direction will the airplane fly to reach an altitude of 20,000 ft above the takeoff point?

83. Connecting Algebra and Geometry Consider the collection of all rectangles that have length 1 cm more than three times their width w .

(a) Find the possible widths (in cm) of these rectangles if their perimeters are less than or equal to 150 cm.

(b) Find the possible widths (in cm) of these rectangles if their areas are greater than 1500 cm².